J R C T E C H N I C A L R E P O R T S

GEM-E3 Model Documentation

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Preface

The main mission of the Joint Research Centre is to provide EU policies with independent, evidence-based scientific and technical support throughout the whole policy cycle. At JRC-IPTS, the Economics of Climate Change, Energy and Transport (ECCET) Unit provides scientific support with a series of economic modeling tools. Some of them are highly disaggregated, sector-specific bottom-up models, which combine rich techno-economic databases with the best knowledge from relevant scientific disciplines, particularly engineering and economics. Other models have been designed to consistently address the impact of sector-specific policies not only on the respective targeted sectors but also on the rest of the economic system.

The computable general equilibrium model GEM-E3 belongs to this second category, closer to what the profession labels as a multi-purpose macroeconomic model. This model has been used in a large set of climate policy applications supporting Commission policy proposals during the last decade, as well as in other environmental and economic policy areas.

The General Equilibrium Model for Economy-Energy-Environment (GEM-E3) model was developed as a multinational collaboration project, partly funded by the European Commission, DG Research, and by national authorities. The model is the result of a large collaborative effort by a consortium of research institutions involving: National Technical University of Athens (NTUA/E3M-Lab) (leading partner), Katholieke Universiteit of Leuven (KUL), University of Manheim, the Centre for European Economic Research (ZEW), and the École Centrale de Paris (ERASME) as the core modeling team. Other contributing teams include PSI, IDEI (University of Toulouse), Stockholm School of Economics, CORE, CEA and University of Strathclyde. Since its first version NTUA/E3M-Lab and KUL have maintained and further developed the GEM-E3 model in various aspects. JRC-IPTS also operates the model and has funded model developments since 2003.

The main purpose of this publication is to provide extensive documentation of the model's equations and its underlying databases, in order to offer to the broader audience an accurate description of the model characteristics. The aim is not only to describe the main model features with the purpose of gaining transparency and credibility, but also the search for feedback and open discussion on the model’s key formulation and main assumptions.

Further information on the model and its policy applications can be found at the model web site: **http://www.gem-e3.net/**).

Antonio Soria
Head of ECCET Unit

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1 History of the GEM-E3 model

The General Equilibrium Model for Economy-Energy-Environment [1] (GEM-E3) model has been developed as a multinational collaboration project, partly funded by the European Commission[2], DG Research, 5th Framework programme and by national authorities, and further developments are continuously under way through several EU Framework Programmes. The model is the result of a collaborative effort by a consortium involving: National Technical University of Athens (NTUA/E3M-Lab) (leading partner), Katholieke Universiteit of Leuven (KUL), University of Manheim and the centre for European Economic Research (ZEW), Ecole Centrale de Paris (ERASME) as the core modelling team. Other contributing teams include PSI, IDEI (University of Toulouse), Stockholm School of Economics, CORE, CEA and University of Strathclyde. The first version of the GEM-E3 model included 12 EU member states and its base year was 1985. Since its first version NTUA/E3M-Lab and KUL have maintained and further developed the GEM-E3 model in various aspects including the introduction of market imperfections, the construction of GEM-E3 world version, bottom-up representation of power generation technologies, equilibrium unemployment, a complete coverage of all GHG, and the introduction of semi-endogenous growth[3] features.

Applications of the model have been carried out for several Directorate Generals of the European Commission (economic affairs, competition, environment, taxation, research) and for national authorities. GEM-E3 is used regularly to provide analytical support to European Commission services, particularly with regards to the economics of climate change.

[1] Informations about the model can also be found in www.e3mlab.ntua.gr and in www.gem-e3.net

[2] JOULE programme, (DG-XII/F1)

[3] This version was built from ERASME, Zagame P. et. Al. « Endogenous Technical Change in GEM-E3 - A Concrete Proposal Involving Spillovers Effects », *Document de travail Erasme*, July 1998

2 Introduction

The GEM-E3 model is a multi-regional, multi-sectoral, recursive dynamic computable general equilibrium (CGE) model which provides details on the macro-economy and its interaction with the environment and the energy system. It is an empirical, large scale model, written entirely in structural form. GEM-E3 allows for a consistent comparative analysis of policy scenarios since it ensures that in all scenarios, the economic system remains in general equilibrium. In addition it incorporates micro-economic mechanisms and institutional features within a consistent macro-economic framework and avoids the representation of behaviour in reduced form. Particularly valuable are the insights the model provides regarding the distributional aspects of long-term structural adjustments. The GEM-E3 model is extensively used as a tool of policy analysis and impact assessment.

The model is modularly built allowing the user to select among a number of alternative closure options and market institutional regimes depending on the issue under study. The GEM-E3 model includes projections of: full Input-Output tables by country/region, national accounts, employment, balance of payments, public finance and revenues, household consumption, energy use and supply, GHG emissions and atmospheric pollutants.

The present document is a manual that illustrates the theoretical foundations on which the model was built by providing mathematical derivations for all the equations included in the model. Moreover, the manual provides the information required to perform different simulations and lists the different options the model offers regarding the model closure and the activation of certain features in the environmental module.

The remainder of this manual is organized as follows: Chapter 2 provides a general presentation of the model and its use for policy analysis, Chapter 3 presents the main data structure of the model, the data sources, the data process manipulation and the GEM-E3 SAM, Chapter 4 provides the mathematical statement of the model, Chapter 5 deals with the welfare measures used in the model, Chapter 6 describes the routine used to make the reference scenario of the GEM-E3 model, Chapter 7 presents the calibration method for the main parameters of the model.

Annexes I-III provide greater details on the model's national accounting concepts and baseline values for energy efficiency and elasticities. Annexes IV-XI describe the following thematic features of GEM-E3: labour market & equilibrium unemployment, environment and emissions module, bottom-up representation of the electricity sector, energy efficiency, energy security indicators, and the stochastic version of the model.

2.1 General model presentation

The world version of the GEM-E3 model simultaneously represents 38 regions and 31 sectors[4] linked through endogenous bilateral trade flows. The model features perfect competition market regimes, discrete representation of power producing technologies, semi-endogenous learning by doing effects, equilibrium unemployment, option to introduce energy efficiency standards, formulates emission permits for GHG and atmospheric pollutants. The environmental module includes flexibility instruments allowing for a variety of options when simulating emission abatement policies, including: different allocation schemes (grandfathering, auctioning, etc.), user-defined bubbles for traders, various systems of exemptions, various systems for revenue recycling, etc.

Its scope is general in two terms: it includes all simultaneously interrelated markets and represents the system at the appropriate level with respect to geography, the sub-system (energy, environment, economy) and the dynamic mechanisms of agent's behaviour.

It formulates separately the supply or demand behaviour of the economic agents which are considered to optimise individually their objective while market derived prices guarantee global equilibrium, allowing the consistent evaluation of distributional effects of policies.

It considers explicitly the market clearing mechanism and the related price formation in the energy, environment and economy markets: prices are computed by the model as a result of supply and demand interactions in the markets and different market clearing mechanisms, in addition to perfect competition, are allowed.

The model formulates production technologies in an endogenous manner allowing for price-driven derivation of all intermediate consumption and the services from capital and labour. In the electricity sector a bottom up approach is adopted for the representation of the different power producing technologies. For the demand-side the model formulates consumer behaviour and distinguishes between durable (equipment) and consumable goods and services.

The model is dynamic, recursive over time, driven by accumulation of capital and equipment. Technology progress is explicitly represented in the production function, either exogenous or endogenous, depending on R&D expenditure by private and public sector and taking into account spillovers effects. Moreover it is based on the myopic expectations of the participant agents[5].
The design of GEM-E3 model has been developed following four main guidelines:

4 The regional and sectoral listing of the model can be found in the ANNEX

5 The model extensions to represent market imperfections and economies of scale were carried out by the National Technical University of Athens (coordinator), the Catholic University of Leuven and Middlesex University.

- Model design around a basic general equilibrium core in a modular way so that different modelling options, market regimes and closure rules are supported by the same model specification.
- Fully flexible (endogenous) coefficients in production and in consumer's demand.
- Calibration to a base year data set, incorporating detailed Social Accounting Matrices as statistically observed.
- Dynamic mechanisms, through the accumulation of capital stock.

The GEM-E3 model starts from the same basic structure as the standard World Bank models[6]. Following the tradition of these models, GEM-E3 is built on the basis of a Social Accounting Matrix (SAM). Technical coefficients in production and demand are flexible in the sense that producers can alternate the mix of production not only regarding the primary production factors but also the intermediate goods. Production is modelled through KLEM (capital, labour, energy and materials) production functions involving many factors (all intermediate products and three primary factors –capital, natural resources and labour). At the same time consumers can also endogenously decide the structure of their demand for goods and services. Their consumption mix is decided through a flexible expenditure system involving durable and non-durable goods. The specification of production and consumption follows the generalised Leontief type of models[7] as initiated in the work of D. Jorgenson (1984).

The GEM-E3 model is built in a modular way around its central CGE core. It supports defining several alternative regimes and closure rules without having to re-specify or re-calibrate the model. The most important of these options are presented below:

- Capital mobility across sectors and/or countries
- Flexible or fixed current account (with respect to the foreign sector)
- Flexible or fixed labour supply
- Market for pollution permits national/international, environmental constraints
- Fixed or flexible public deficit
- Perfect competition or Nash-Cournot[8] competition assumptions for market competition regimes

The model is not limited to comparative static evaluation of policies. The model is dynamic in the sense that projections change over time. Its properties are mainly manifested

[6] The World Bank type of models constitutes the major bulk of equilibrium modelling experiences. This type of models was usually used for comparative statics exercises. The World Bank and associated Universities and scientists have animated a large number of such modelling projects, usually applied to developing countries. Main authors in this group are J. De Melo, S. Robinson, R. Eckaus, S. Devarajan, R. Decaluwe, R. Taylor, S. Lusy and others. These models however do not use full scale production functions but rather work on value added and their components to which they directly relate final demand

[7] The generalised Leontief type of model was first formulated empirically in the work of D. W. Jorgenson who introduced flexibility in the Leontief framework, using production functions such as the translog. The work of D. W. Jorgenson inspired many modelling efforts, in which particular emphasis has been put to energy. For example, such models have been developed in France, by P. Capros, N. Ladoux, in OECD (GREEN and WALRAS), in Sweden by L. Bergman and in Germany by K. Conrad.

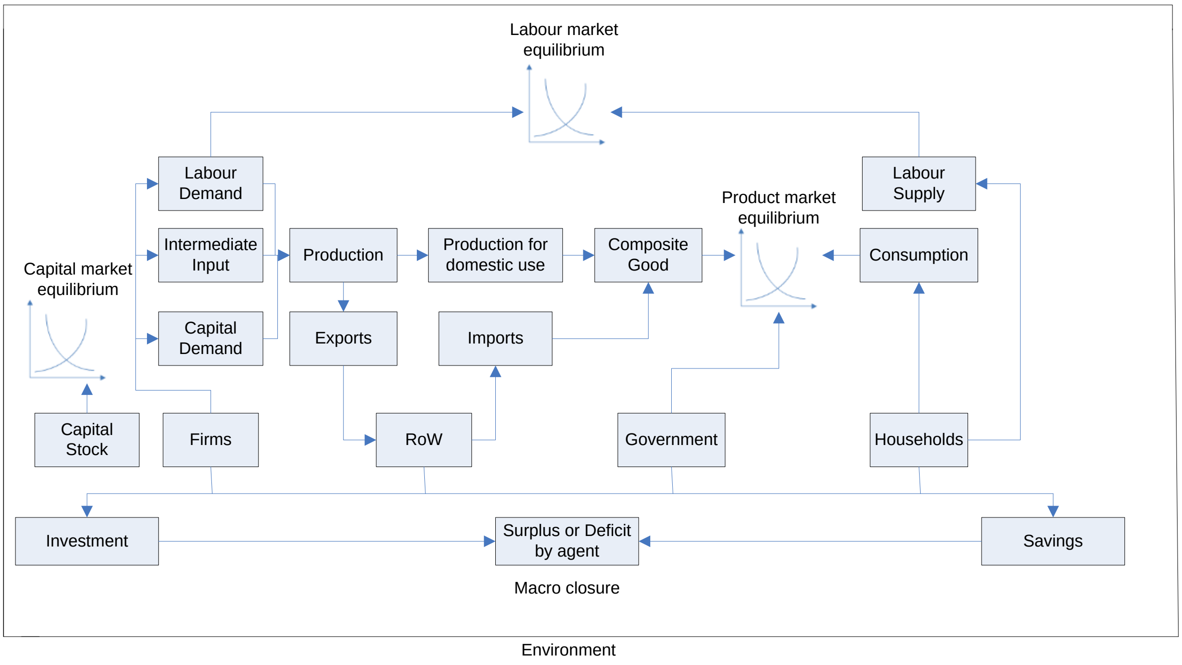
[8] This option is available only for the EU version of the GEM-E3 model

through stock/flow relationships, technical progress, capital accumulation and agents' (myopic) expectations.

The model is calibrated to a base year data set that comprises a full Social Accounting Matrices for each country/region represented in the model. Bilateral trade flows are also calibrated for each sector represented in the model, taking into account trade margins and transport costs. Consumption and investment is built around transition matrices linking consumption by purpose to demand for goods and investment by origin to investment by destination. The initial starting point of the model therefore, includes a very detailed treatment of taxation and trade.

Total demand (final and intermediate) in each country is optimally allocated between domestic and imported goods, under the hypothesis that these are considered as imperfect substitutes (the "Armington" assumption[9]).

Figure 1: GEM-E3 economic circuit



Institutional regimes, that affect agent behaviour and market clearing, are explicitly represented, including public finance, taxation and social policy. The model represents goods that are external to the economy as for example damages to the environment.

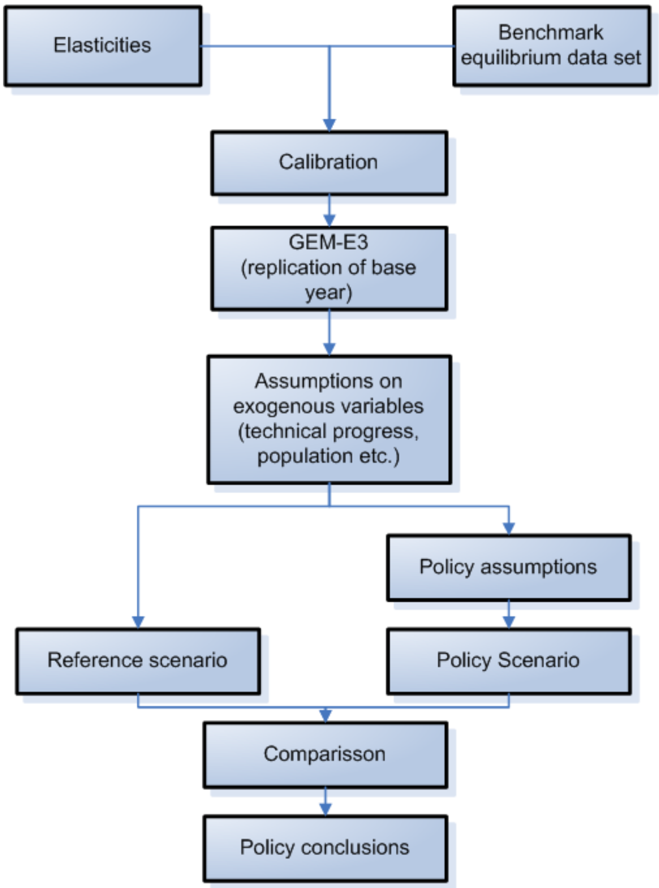
Figure 1 illustrates the overall structure of the GEM-E3 model.
The internalisation of environmental externalities is achieved either through taxation or global system constraints, the shadow costs of which affect the decision of the economic agents. In the *GEM-E3* model global/regional/sectoral constraints are linked to environmental emissions, changes in consumption or production patterns, external

[9]See Armington (1969).

costs/benefits, taxation, pollution abatement investments and pollution permits. The model evaluates the impact of policy changes on the environment by calculating the change in emissions and damages and determines costs and benefits through an equivalent variation measurement of global welfare (inclusive environmental impact).

2.2 Counterfactual simulations

Once the model is calibrated, the next step is to define a reference case scenario. The reference case scenario includes all already decided policies. The key drivers of economic growth n the model are labour force, total factor productivity and the expectations on sectoral growth. The "counterfactual" equilibria can be computed by running the model under assumptions that diverge from those of the reference scenario. This corresponds to scenario building. In this case, a scenario is defined as a set of changes of exogenous variables, for example a change in the tax rates. Changes of institutional regimes, that are expected to occur in the future, may be reflected by changing values of the appropriate elasticities and other model parameters that allow structural shifts (e.g. market regime). These changes are imposed on top of the assumptions of the reference scenario thereby modifying it. To perform a counterfactual simulation it is not necessary to re-calibrate the model. The different steps for performing a counterfactual simulation in GEM-E3 are depicted in the figure above.



A counterfactual simulation is characterised by its impact on consumer's welfare or through the equivalent variation of his welfare function. The equivalent variation can be, under reasonable assumptions, directly mapped to some of the endogenous variables of the model such as consumption, employment and price levels. The sign of the change of the equivalent variation gives then a measure of the policy's impact and burden sharing implications. The most important results, provided by GEM-E3, are as follows:

- Dynamic annual projections in volume, value and deflators of national accounts by country.
- Full Input-Output tables for each country/region identified in the model
- Distribution of income and transfers in the form of a social accounting matrix by country.
- Employment, capital, investment by country and sector.
- Greenhouse gasses, atmospheric emissions, pollution abatement capital, purchase of pollution permits and damages.

- Consumption matrix by product and investment matrix by ownership branch.
- Public finance, tax incidence and revenues by country.
- Full bilateral trade matrices.

2.3 Solution algorithm

The model is formulated as a simultaneous system of equations with an equal number of variables. The system is solved for each year following a time-forward path. The model uses the GAMS software and is written as a mixed non-linear complementarity problem solved by using the PATH algorithm using the standard solver options.

2.4 Policy Analysis Support

The GEM-E3 model has been extensively used by several DGs of the European Commission for policy analysis. GEM-E3 is a general-purpose model that aims to cope with the specific orientation of the policy issues that are actually considered at the level of the European Commission. Policies are analysed as counterfactual dynamic scenarios and are compared against reference model runs. Policies are then evaluated through their impact on sectoral growth, finance, income distribution and global welfare.

The GEM-E3 model intends to cover the general subject of **sustainable economic growth**, and to support the study of related policy issues. Sustainable economic growth is considered to depend on combined environmental and energy strategies that will ensure stability of economic development. The general issue, to be analysed with GEM-E3, regards the conditions under which economic growth, and its distributional pattern, can be sustained in the presence of environmental constraints or energy shortages and even reinforced by means of an adequate technological and market-oriented policy.

The model intends, in particular, to analyse the **global climate change** issue a theme that embraces several aspects and interactions within the economy, energy and environment systems. To reduce greenhouse gas emissions it is necessary to achieve substantial gains in energy conservation and in efficiency in electricity generation, as well as to perform important fuel substitutions throughout the energy system, in favour of less carbon intensive energy forms.

Moreover, within the context of increasingly competitive markets, new policy issues arise. For example, it is necessary to give priority to market-oriented policy instruments, such as carbon taxes and pollution permits, and to consider market-driven structural changes, in order to maximise effectiveness and alleviate macroeconomic consequences. Re-structuring of economic sectors and re-location of industrial activities may be also induced by climate change policies. This may have further implications on income distribution, employment, public finance and the current account.

The model is designed to support the analysis of **distributional effects** that are considered in two senses: distribution among countries and distribution among social and economic groups within each country. The former issues involve changes in the allocation of capital, sectoral activity and trade and have implications on public finance and the current account of member states. The assessment of allocation efficiency of policy is often termed "**burden sharing analysis**", which refers to the allocation of efforts (for example taxes), over different countries and economic agents. The analysis is important to adequately define and allocate compensating measures aiming at maximizing economic cohesion. Regarding both types of distributional effects, the model can also analyse and compare coordinated versus non coordinated policies in the European Union.

Technical progress and infrastructure can convey factor productivity improvement to overcome the limits towards sustainable development and social welfare. For example, European RTD strategy and the development of pan-European infrastructure are conceived to enable long-term possibilities of economic growth. The model is designed to support analysis of structural features of economic growth related to technology and evaluate the derived economic implications for competitiveness, employment and the environment.

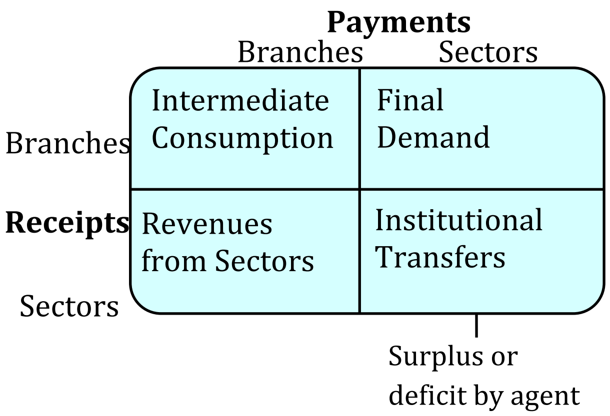
The model puts emphasis on:

- The analysis of market instruments for energy-related environmental policy, such as taxes, subsidies, regulations, emission permits etc., at a degree of detail that is sufficient for national, sectoral and World-wide policy evaluation.
- The assessment of distributional consequences of programmes and policies, including social equity, employment and cohesion for less developed regions.
- The standard need of the European Commission to periodically produce detailed economic, energy and environment policy scenarios.

3 The GEM-E3 Database

3.1 Social Accounting Matrix (SAM)

After F. Quesnay's (1759) tableaux economique it was the Leontief input-output tables that attempted to describe the structural form and the interdependencies among the agents of an economic system. The need to synthesize the economic and the social dimension of these interdependencies led to the extension of the Leontief's IO table into the Social Accounting Matrix (SAM). These matrices are essentially a statistical methodological framework that allows the systematic recording of: i) the way that goods and services are produced and consumed and ii) the creation and distribution of income amongst the different economic agents. SAMs are square matrices of monetary flows that describe all the money transactions that take place among the economic agents within a certain period of time (usually a year). The number of the agents defines the dimensions of the matrix.



The columns of the matrix represent expenditures and the rows represent receipts. Expenditures equal receipts for each commodity. By construction the SAMs satisfy Walras law (the excess demand of all economic agents is zero). The SAM on which the GEM-E3 model is based, is presented in Figure 2. The GEM-E3 SAM is expressed at producer prices[10]. The construction of the SAM is the starting point of the model building work. The SAMs of the world version of the GEM-E3 model are based on the GTAP database, whereas for the European version, the symmetric input-output tables and national accounts from EUROSTAT are used.

The SAM of GEM-E3 represents flows between production sectors, production factors and economic agents. The production sectors produce an equal number of distinct goods (or services), as in an Input-Output table. The SAM distinguishes between intermediate and primary production factors. The economic agents, namely households, firms, government and the foreign sector, are owners of the primary production factors, so they receive income from labour and capital rewarding. All interinstitutional transactions amongst the different agents as recorded in the national accounts are captured by the SAM. The agents use part of their income for consumption and investment, and form final domestic demand. The foreign sector also makes transactions with each other sector. These transactions

[10] Producer prices: all taxes on production are included (VAT and transportation cost are not included), purchaser price: Producer prices + transportation cost.

represent imports (as a row) and exports (as a column) of goods and services. The difference between income and spending (in consumption and investment) by an economic agent determines his surplus or deficit.

Figure 2: GEM-E3 Social Accounting Matrix

	01…………. …………...n	Total intermediate demand	Labour	Capital	Total	Household consumption incl. NPISHs)	Public consumption	Firms	Investments	Change in Stocks	Exports	Total final demand	Total Demand
01 **n**	IO	**[8]**				**[9]** HC	**[10]** GC		**[11]** INV	**[12]** STV	**[13]** EXP	**[14]=[9]+[10]+ [11]+[12]+[13]**	**[15]=[14]+[8]**
Total intermediate inputs	**[1]**												
Operating surplus	KA										FFASE		
Wages and Salaries	LA										FFASE		
Social security contribution	SS												
Total Value Added	**[2]**												
Total supply at basic prices	**[3]=[1]+[2]**												
Households			FSEFA: labour income	FSEFA: income from operating surplus		FSESE	FSESE: (i.e. social benefits, pension)	FSESE			FSESE		
Firms				FSEFA: income from operating surplus		FSESE	FSESE	FSESE			FSESE		
VAT	TX_VAT*(vat_base)												
Subsidies	TX_SUB*(sub_base)							FGRS					
Direct taxes						FGRS: (i.e. income from direct taxation)		FGRS			FGRS		
Social security contributions						FGRS							
Indirect taxes	TX_IT*(it_base)					FGRS					FGRS		
Duties	TX_DUT*(dut_base)												
Environmental taxes	TX_ENV*(env_base)												
Government - Firms				FGRF		FGRS		FGRS			FGRS		
Government - Rest of the World											FGRS		
Total taxes	**[4]**												
Total supply at producer prices	**[5]=[4]+[3]**												
Imports	**[6] IMP**		FSEFA	FSEFA		FSESE	FSESE	FSESE					
Savings						SAVE	SAVE	SAVE			SAVE		
Total supply	**[7]=[5]+[6]**												

Demand = Supply **[7] = [15]**

FSESE: Transactions from sector to sector
FSEFA: Transactions from factors to sector
FGRF: Government ownerhsip of firms
FGRS: Transactions of government with the other economic agents

In the GEM-E3 model firms are modelled to maximize their profits, constrained by the physical capital stock (fixed within the current period) and the available technology. Producers can change their physical capital stock over time through investment. Capital stock data by sector of production are not available either from GTAP or from EUROSTAT databases (it is computed in the calibration phase of the model).

Households in the GEM-E3 SAM are identified as a single social group (a single representative household is modelled). Households maximize their inter-temporal utility under an inter-temporal budget constraint. The demand functions are derived by solving the maximization problem, under general assumptions regarding expectations and steady state conditions. These demand functions allocate the expected income of the household, depending on the formulation of the problem, between consumption goods and future consumption (savings). This is the default formulation of households' behaviour alternatively household behaviour is modelled so that the consumer allocates its expected income between present, future consumption and leisure. For household consumption, the model considers an allocation mechanism. The allocation mechanism considers durable and non-durable goods. Durable goods include cars, heating systems and electric appliances, and their use involves demand for non-durable goods, mainly energy (fuels and electricity).

3.2 Institutional accounts

The GEM-E3 model has a detailed representation of all institutional transactions between economic agents. The Institutional sectors according to the national accounts classification are presented in Table 1. The institutional accounts cover all the transactions between the institutional sectors of the economy.

Table 1: Institutional sectors of the economy

Sectors	Description
SS	All Sectors
S11	Non-financial corporations
S12	Financial corporations
S13	General government
S14_S15	Households; non-profit institutions serving households
S2	Rest of the world

In order to build the table “transactions between sectors” (FSESE - Figure 2) two sets of tables are required: i) The full set of tables for the full sequence of accounts of each institutional sector, ii) A matrix presentation of the most important transactions of the system. A matrix presentation permits each transaction to be represented by a single entry and the nature of transaction to be inferred from its position.

Table 2: D5 Taxes on Income – Germany

	H	F	G	W	TOTAL
H					0
F					0
G	217040	34590	0	2430	254060
W			110		110
TOTAL	217040	34590	110	2430	254170

Each transaction between two institutional sectors is represented by a column and a row pair. The convention followed is that resources are shown in the rows and uses are shown in the columns. For instance (see Table 2), taxes on income (D5) are payable by the Households and received by the government.

The institutional transactions are grouped in two main categories: current account transactions and accumulation accounts. The current account and its different components as defined by the ESA 95 are presented below:

The production account which refers to all transactions related to production (balancing item: Gross value added).

Received (Resources)	Paid (Uses)
Output	Intermediate consumption
Gross Value Added	Taxes less subsidies on products

The generation of income account which shows how the proceeds of the production are allocated to the various income categories (balancing item: mixed income/gross operating surplus).

Received (Resources)	Paid (Uses)
Compensation of Employees	Net Value Added
Other Taxes on Production	Other Subsidies on production

The allocation of primary income account which shows receipts and expenditures related to various forms of property income such as interests, dividends, rents, (balancing item: balance of primary incomes)

Received (Resources)	Paid (Uses)
Property income	Net operating surplus
Interest	Property income
Distributed income of corporations	Interest
Dividends	Distributed income of corporations
Withdrawals from income of quasi-	Dividends
Net balance of primary incomes/Net	Withdrawals from income of quasi-corporations
	Net balance of primary incomes/Net national income

The secondary distribution of income account shows how the primary income of an institutional sector changes because of current taxes on income and wealth, social contributions and benefits, and other current transfers. The balancing item is disposable income.

Received (Resources)	Paid (Uses)
Current taxes on income, wealth, etc.	Net balance of primary incomes/Net national
Taxes on income	Social contributions
Other current taxes	Actual social contributions
Social benefits other than social	Employers' actual social contributions
Other current transfers	Other current transfers
Net non-life insurance premiums	Net non-life insurance premiums
Non-life insurance claims	Non-life insurance claims
Current transfers within general	Current transfers within general government
Current international cooperation	Current international cooperation
Miscellaneous current transfers	Miscellaneous current transfers
Net disposable income	

The use of disposable income account shows how disposable income is spent on consumption or saved (The balancing item is saving)

Received (Resources)	Paid (Uses)
Adjustment for the change in net equity	Net disposable income
Net saving	

The external account brings together all transactions involving both euro area residents and non-residents, viewed from the perspective of the non-residents.

Received (Resources)	Paid (Uses)
Exports of goods and services	Imports of goods and services
External balance of goods and services	

The capital account is an accumulation account. It is divided into a change in net worth due to saving and capital transfers account and an acquisition of non-financial assets account. The first adds any net receipts of capital transfers to net saving. The balancing item is the change in net worth due to transactions. The acquisition of non-financial assets account records gross fixed capital formation (investment in non-financial assets), changes in inventories, and any net acquisition of valuables and other non-produced, non-financial assets (e.g. land). The balancing item of the capital account is net lending/net borrowing.

Received (Resources)	Paid (Uses)
Capital transfers	Net saving
Capital taxes	Capital transfers
Investment grants	Capital taxes
Other capital transfers	Investment grants
Changes in net worth due to saving and capital	Other capital transfers

In the following we provide the exact computation (through the national accounts) of the GEM-E3 transfers. The main classifications of the ESA-95 national accounts are presented in Table 3.

Table 3: National accounts – institutional transfers categories

B1G	Gross value added (at basic prices)
B2G_B3G	Gross operating surplus and gross mixed income
B5G	Gross national income/Balance of primary incomes, gross
B6G	Gross disposable income
B8G	Gross saving
B11	External balance of goods and services
B101	Changes in net worth due to saving and capital transfers
B9	Net lending (+) /net borrowing (-)
P1	Output
P2	Intermediate consumption
P3	Final consumption expenditure
P5	Gross capital formation
P6	Exports of goods and services
P7	Imports of goods and services
D1	Compensation of employees
D2	Taxes on production and imports
D21	Taxes on products
D29	Other taxes on production
D3	Subsidies
D31	Subsidies on products
D39	Other subsidies on production
D4	Property income
D5	Current taxes on income, wealth, etc.
D6	Social contributions and benefits
D61	Social contributions
D62	Social benefits other than social transfers in kind
D63	Social transfers in kind
D7	Other current transfers
D8	Adjustment for the change in net equity of households in pension funds reserves
D9	Capital transfers
K1	Consumption of fixed capital
K2	Acquisitions less disposals of non-financial non-produced assets

A detailed representation of the arrangement of national accounts can be found in Annex I.

3.3 Consumption matrix

The consumption matrix decomposes the demand per consumer categories (COICOP, the list of the GEM-E3 consumer categories is found in Annex III) into deliveries by sector of production. These matrices are usually reported in consumer's prices (ESA 95 valuation concept), i.e. VAT and margins are included in the price of the delivery and moreover margins are not considered as a separate delivery by a service branch. In the GEM-E3 model, this matrix is transformed in producer's prices.

Therefore, the following corrections are made:

i) given the VAT rates for the different consumer categories, a consumption matrix without VAT is computed,

ii) the margins included in the deliveries by branch are evaluated as the difference between the consumption matrix deliveries (without VAT) and the IO deliveries,

iii) margins are allocated between the services branches.

Table 4 presents the consumption matrix coefficients of UK in the GEM-E3 product classification (as % shares in total consumption).

Table 4: UK Consumption matrix coefficients

	FOOD BEVERAGES AND TOBACCO	CLOTHING AND FOOTWEAR	HOUSING AND WATER CHARGES	FUELS AND POWER	HOUSEHOLD EQUIPMENT AND	HEATING AND COOKING APPLIANCES	MEDICAL CARE AND HEALTH	PURCHASE OF VEHICLES	OPERATION OF PERSONAL TRANSPORT	TRANSPORT SERVICES	COMMUNICATION	RECREATIONAL SERVICES	MISCELLANEOUS GOODS AND SERVICES	EDUCATION
Agriculture	9.7			0.1	0.2							1.5		
Coal				2.4										
Crude Oil														
Oil				5.8					55.7					
Gas				22.7										
Electricity supply				69.0										
Ferrous metals														
Non-ferrous metals			0.6		4.2	1.2			0.1				0.5	
Chemical Products			3.1		11.2		30.2		2.0			0.6	9.3	
Paper Products			1.0		7.4							0.1	13.1	
Non metallic minerals			0.3		5.1								0.1	
Electric Goods					20.7	0.1			1.9		3.4			
Transport equipment					7.0			81.6	3.6					
Other Equipment Goods			0.8		22.5	87.2	21.8					6.7	7.1	
Consumer Goods Industries	90.1	98.0	0.9		15.7							2.1	0.6	
Construction			6.3											
Transport (Air)										42.8				
Transport (Land)										48.9				
Transport (Water)										6.2				
Market Services	0.2	2.0	83.8		5.9	11.5	2.6	18.4	33.1	1.4	96.6	10.4	58.2	
Non Market Services			3.3				45.4		3.5	0.7		78.5	11.0	100.0
Total	100	100	100	100	100	100	100	100	100	100	100	100	100	100

In the cases where consumption matrices are not available from statistical sources, they are computed through the following way:

i. The consumption per consumer category (COICOP) is extracted from the National-Accounts (final consumption of households on the economic territory, by purpose) and corrected for the consumption by tourist,
ii. Given the VAT rates for the different consumer categories, the total consumption per category without VAT is computed,
iii. Total deliveries are taken from the Input-Output tables,
iv. Once the row and columns totals of the consumption matrix are computed for each country/region a RAS procedure is applied (the initial coefficients for the RAS are taken from countries with available consumption matrices).

3.4 Investment matrix

The investment matrix decomposes investment by sector of production into deliveries by branches. Hence the row total represents the consumption of fixed capital found in the IO tables and the column total represents the investment each firm performs within a year. Data regarding investments in power generation technologies have been extracted from JEDI[11] and EWEA (2009). In the GEM-E3 model, investments are computed:

i. by applying the uniform investment coefficients based on gross fixed capital formation found in the IO tables and by adding additional data for specific branches (where available),
ii. by applying a RAS method in order to ensure that the investment shares are in line with the consumption of the fixed capital.

[11] See: http://www.nrel.gov/analysis/jedi/about_jedi.html

Table 5: Deliveries of branches to firm investment for selected countries

	Germany	Spain	France	UK	USA	Japan	China	India
Agriculture	0.39	0.20	0.14	0.17		0.07	1.21	0.20
Ferrous metals	0.07							2.11
Non-ferrous metals	3.05	2.32	1.27	3.38	0.57	0.48	0.90	4.09
Chemical Products	0.15	0.03		0.29	0.12		0.03	1.02
Paper Products	0.17			0.35				
Non-metallic minerals	0.04	0.01	0.01	0.16	0.01	0.01		0.36
Electric Goods	8.80	5.04	1.24	9.57	6.98	10.50	4.69	3.52
Transport equipment	10.90	9.28	4.72	9.53	12.59	5.06	9.03	7.77
Other Equipment Goods	23.37	14.34	7.07	16.76	16.62	14.52	21.92	24.77
Consumer Goods Industries	1.09	0.14	0.03	0.30	3.01	0.32	1.12	0.06
Construction	41.04	52.06	58.16	45.47	45.72	52.93	53.46	48.62
Transport (Air)					0.31		0.05	0.29
Transport (Land)	0.08	0.09	0.09	0.43	0.78	0.60	0.19	2.47
Transport (Water)				0.15	0.14	0.04	0.09	0.03
Market Services	9.96	15.92	26.37	12.01	13.14	15.45	6.97	4.70
Non Market Services	0.88	0.56	0.89	1.41			0.31	
Total	100	100	100	100	100	100	100	100

From

Table **5** it is clear that the investment deliveries are basically made by the branches of: Electric Goods, Transport Equipment, Other Equipment Goods Industries, Market services and Construction with the latter having the largest share in the deliveries for investment among all branches.

3.5 Labour market data

The following data are essential for the modelling of GEM-E3 labour market:

i. Skilled and unskilled labour force (total and by category),
ii. Unemployment rate for skilled and unskilled labour force.

The GEM-E3 model adopts the EUROSTAT definition of the labour force and thus it is computed by multiplying the participation rate to total active population. The databases mainly used to extract these data are the EUROSTAT, ILO and World Bank.

3.6 Bilateral Trade

Regarding foreign trade data, the GEM-E3 model requires detailed bilateral trade matrices for all regions and commodities included in the model. GTAP database provides such matrices together with bilateral duties and transportation costs. For countries that are not identified separately in GTAP the UN Comtrade database is used in order to extract the relevant data.

3.7 GHG emissions

The GEM-E3 model covers the following greenhouse gasses: CO_2, CH_4, N_2O, HFCs, PFs and SF_6. In the model these emissions are linked to the activity level of the relevant sectors. This link is presented in Table 6. Data on GHG emissions are extracted from the UNFCCC database and estimates for process related GHG MACCs are taken from "Global mitigation of non-CO2 GHG" EPA report (2006), and IIASA database.

Table 6: GHG emission sources and link with GEM-E3 activities

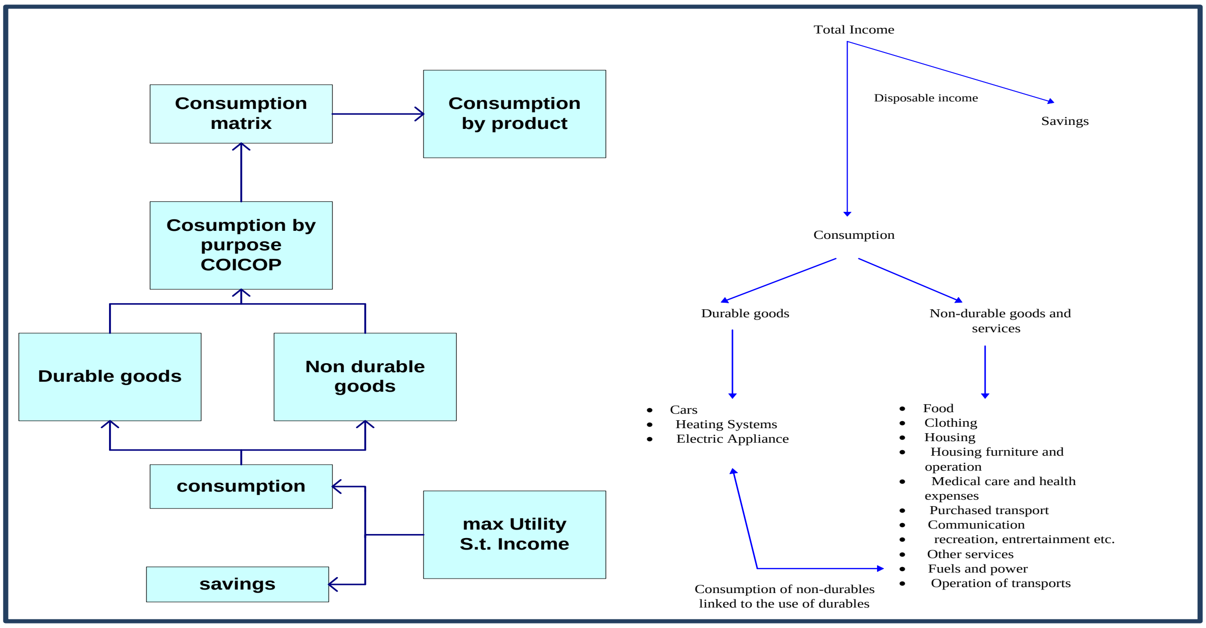
GHG	Sources	GEM-E3 activity	% in total GHG emissions of Annex-I (2005)	GWP
CO_2	Burning of fossil fuels	All activities	0,785	1
CO_2	Cement, chemicals and non-metallic minerals production[12]	Ferrous metals, chemical products and non-metallic minerals	0,04	1
CH_4	Waste management, Gas and Coal mining, Oil, Animals	Mainly Coal, Oil, Gas, Chemical products, Transport and Non Market services	0,12	24
N_2O	Burning of fossil fuels, Transport, Production of adipic and nitric acid (nylon), Fertilisers	Mainly Agriculture, Oil, Electricity transmission and distribution, Transport, Chemical products	0,057	310
HFC	CFC substitute, Production of HCFC-22, refrigerators	Chemical products, Electric goods	0,0119	2000
PFC	Production of aluminium, semiconductors	Ferrous metals, Electric goods	0,002	6800
SF_6	Magnesium production, power distribution, Production of aluminium	Power supply, Ferrous metals	0,002	22200

4 Mathematical model statement

4.1 Household behaviour

Households receive income from their ownership of production factors, from other institutions and transfers from the rest of the world. Household expenditure is allocated between consumption, tax payment and savings. The representative household firstly decides on the allocation of its income between present and future consumption of goods. At a 2nd stage the household allocates its total consumption expenditure between the different consumption categories available. The consumption categories are split in non-durable consumption categories (food, culture etc.) and services from durable goods (cars, heating systems and electric appliances). The general form that is described above is being depicted with a nesting scheme as it is appeared below.

Figure 3: The consumption structure of the GEM-E3 model



The general specification of the 1st stage problem, with a time separable Stone-Geary utility function, can be written as follows:

$$\max U_{er,t} = \sum_t (1 + stp_{er,t})^{-t} \cdot (bh_{er,t} \cdot \ln(HCDTOTV_{er,t} - ch_{er,t})) \qquad [1]$$

where:

$HCDTOTV_{er,t}$: represents the consumption of goods (in volume),

$stp_{er,t}$: the subjective discount rate of the households, or social time preference,

$ch_{er,t}$: the subsistence quantity of consumption,

$bh_{er,tt}$: the share of consumption in the disposable income of the households (equal to unity in the standard version of GEM-E3 where no leisure choice is considered).

The maximisation is subject to the following inter-temporal budget constraint, which states that all available disposable income will be spent either now or sometime in the future:

$$\sum_t (1+r_{er,t})^{-t} \cdot (HCDTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t}) = \sum_t (1+r_{er,t})^{-t} \cdot (YTR_{er,t} - PCI_{er,t} \cdot ch_{er,t}) \qquad [2]$$

where:

$r_{er,t}$: discount rate,

$HCDTOT_{er,t}$: total private consumption,

$PCI_{er,t}$: consumer price index,

$YTR_{er,t}$: total available income of the households from all sources

The non-wage income is income such as interest payments from assets, share in firms' profits, social benefits, and remmitances. Based on myopic assumptions about the future, the household decides the desired amount of income. For a given time *t*, the budget constraint becomes:

$$\mathrm{YTR_{er,t}} = \mathrm{YDISP_{er,t}} \qquad [3]$$

where:
$YDISP_{er,t}$: the disposable income,

Equation [3] states that at a given period in time the sum of the total income and the value of the household's time endowment will be equal to the income available for consumption and savings. Under myopic expectations, the values of the right hand-side in equation [2] are assumed to increase at a constant rate f (say, according to the wage rate). Then the r.h.s of the equation, combined with equation [3], for a given year (for example *t=0*) becomes:

$$\sum_t (1+r_{er,t})^{-t} \cdot (1+f_{er,t})^{t} \cdot (YDISP_{er,0} - PCI_{er,0} \cdot ch_{er,t}) = (YDISP_{er,0} - PCI_{er,0} \cdot ch_{er,t}) \cdot \sum_t \left(\frac{1+r_{er,t}}{1+f_{er,t}}\right)^{-t} = (YDISP_{er,0} - PCI_{er,0} \cdot ch_{er,t}) \cdot \left(\frac{1}{rr_{er,t}}\right) = \left(\frac{1}{rr_{er,t}}\right) \cdot IC_{er,t} \qquad [4]$$

Where:
$rr_{er,t}$: the real interest rate. In the GEM-E3 model it is defined as $rr_{er,t} = RLTLREU_t \cdot RLTLR_t \cdot RLTLRWORLD_t$ for the case in which the country is member of the European Union and $rr_{er,t} = RLTLR_t \cdot RLTLRWORLD_t$ for all the other countries.
$IC_{er,t}$: total available income at period t=0.
Equation [4] is the present value of the total income of the household. The factor $\sum_{t=0}^{\infty}\left(\frac{1+r_{er,t}}{1+f_{er,t}}\right)^{-t}$ can be approximated by $\frac{1}{rr_{er,t}}$, that is the inverse of the real discount rate (for a T sufficiently large $\sum_{t=0}^{\infty}\left(\frac{1+r_{er,t}}{1+f_{er,t}}\right)^{-t}$ converges to $\frac{1}{rr_{er,t}}$).

At an arbitrary year the maximization problem of the household is:

$$\max U_{er,t} = \left(1 + stp_{er,t}\right)^{-t} \cdot \left(bh_{er,t} \cdot \ln\left(HCDTOTV_{er,t} - ch_{er,t}\right)\right)$$

such that:

$$\left(1 + r_{er,t}\right)^{-t} \cdot \left(HCDTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t}\right) = \left(1 + r_{er,t}\right)^{-t} \cdot \left(1 + f_{er,t}\right)^{t} \cdot IC_{er,t}$$

The Lagrangian of the above problem is:

$$\mathcal{L} = \left(1 + stp_{er,t}\right)^{-t} \cdot \left(bh_{er,t} \cdot \ln\left(HCDTOTV_{er,t} - ch_{er,t}\right)\right) - \lambda \cdot \left(1 + r_{er,t}\right)^{-t} \cdot \left(HCDTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t} - \left(1 + f_{er,t}\right)^{t} \cdot IC_{er,t}\right) \quad [5]$$

Taking the first order conditions and the budget constraint [2], the derived demand function is obtained:

$$\left\{\left(1 + stp_{er,t}\right)^{-t} \cdot \frac{bh_{er,t}}{\left(HCDTOTV_{er,t} - ch_{er,t}\right)} - \lambda \cdot \left(1 + r_{er,t}\right)^{-t} \cdot PCI_{er,t} = 0 \Rightarrow\right.$$

$$\Rightarrow \left\{HCDTOTV_{er,t} - ch_{er,t} = \frac{1}{\lambda} \cdot \left(\frac{1 + stp_{er,t}}{1 + r_{er,t}}\right)^{-t} \cdot \frac{bh_{er,t}}{PCI_{er,t}}\right. \quad [6]$$

The value of the Lagrangian multiplier λ can be derived by summing up this equation over time, and substituting the demand function into the budget constraint yields:

$$\sum_t \left(1 + r_{er,t}\right)^{-t} \cdot \left(HCDTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t}\right) = \frac{1}{rr} \cdot IC_{er,t} \Rightarrow \sum_t \left(1 + r_{er,t}\right)^{-t} \cdot \left[PCI_{er,t} \cdot \left(HCDTOT_{er,t} - ch_{er,t}\right)\right]$$
$$= \frac{1}{rr} \cdot IC_{er,t} \Rightarrow \sum_t \left(1 + r_{er,t}\right)^{-t} \cdot \left[PCI_{er,t} \cdot \left(\frac{1}{\lambda}\left(\frac{1 + stp_{er,t}}{1 + r_{er,t}}\right)^{-t} \cdot \frac{bh_{er,t}}{PCI_{er,t}}\right)\right] = \frac{1}{rr_{er,t}} \cdot IC_{er,t}$$
$$\Rightarrow \sum_t \left(1 + str_{er,t}\right)^{-t} \cdot \left(bh_{er,t}\right) \cdot \frac{1}{\lambda} = \frac{1}{rr_{er,t}} \cdot IC_{er,t} \Rightarrow \frac{1}{\lambda} = \frac{stp_{er,t}}{rr_{er,t}} \cdot \frac{1}{bh_{er,t}} \cdot IC_{er,t}$$

Expressing now the equation [6] for the current time period (*t=0*) and using the value of the multiplier, the demand function used in the model is obtained:

$$HCDTOTV_{er,t} = \begin{cases} ch_{er,t} + \dfrac{stp_{er,t}}{RLTLREU_t \cdot RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bh_{er,t}}{PCI_{er,t}} \cdot \left(YDISP_{er,t} - OBL_{er,t}\right) & if\ er = euc27 \\ ch_{er,t} + \dfrac{stp_{er,t}}{RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bh_{er,t}}{PCI_{er,t}} \cdot \left(YDISP_{er,t} - OBL_{er,t}\right) & if\ er \neq euc27 \end{cases} \quad [7]$$

]

where:

$OBL_{eu,t} = PCI_{eu,t} \cdot ch_{eu,t}$: the minimum obliged consumption of goods.

Given the fact that the model is calibrated to a base year dataset in which households have a positive savings rate, the computed *stp* is less than *rr*. The savings rate in the above equation is not fixed but rather depends on factors such as the social time preference, the real interest rate and the relative shares of consumption in total disposable income.

In an alternative formulation of the model household allocates its income between present and future consumption of goods and leisure. The utility function can be written as:

$$\max U_{er,t} = \sum_t (1 + stp_{er,t})^{-t} \cdot \left(bh_{eur,t} \cdot \ln(HCDTOTV_{er,t} - ch_{er,t}) + bl_{eur,t} \cdot \ln(LJV_{er,t} - cl_{er,t})\right)$$

where:

$LJV_{er,t}$: represents the consumption of leisure,

$bl_{eur,t}$: the respective shares of leisure in the disposable income of the households

The intertemporal budget constraint is now augmented so as to include the value of the households' time endowment and can be written as:

$$\sum_t (1 + r_{er,t})^{-t} \cdot (HCDTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t} + PLJ_{er,t} \cdot LJV_{er,t} - PLJ_{er,t} \cdot cl_{er,t}) = \sum_t (1 + r_{er,t})^{-t} \cdot (YTR_{er,t} + PLJ_{er,t} \cdot LTOT_{er,t} - PCI_{er,t} \cdot ch_{er,t} - PLJ_{er,t} \cdot cl_{er,t})$$

where:

$PLJ_{er,t}$: the price of leisure,

$LTOT_{er,t}$: total available time to households,

Based on myopic assumptions about the future, the household decides the amount of leisure that wishes to forsake in order to acquire the desired amount of income (thus also defining labour supply behaviour). The budget constraint states that at a given period in time the sum of the total income will be equal to the income available for consumption, plus savings, plus the value of leisure:

$$\mathrm{YTR_{er,t}} + PLJ_{er,t} \cdot LTOT_{er,t} = \mathrm{YDISP_{er,t}} + PLJ_{er,t} \cdot LJV_{er,t}$$

Assuming that the total available income increases at a constant rate (as in the standard problem described above) and solving the maximization problem gives rise to the following first order conditions, namely the demand functions for consumption and leisure:

$$HCDTOTV_{er,t} = \begin{cases} ch_{er,t} + \dfrac{stp_{er,t}}{RLTLREU_t \cdot RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bh_{er,t}}{PCI_{er,t}} \cdot \left(YDISP_{er,t} + PLJ_{er,t} \cdot LJV_{er,t} - OBL_{er,t}\right) & if\ er = euc27 \\ ch_{er,t} + \dfrac{stp_{er,t}}{RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bh_{er,t}}{PCI_{er,t}} \cdot \left(YDISP_{er,t} + PLJ_{er,t} \cdot LJV_{er,t} - OBL_{er,t}\right) & if\ er \neq euc27 \end{cases}$$

$$LJV_{eu,t} = \begin{cases} cl_{eu,t} + \dfrac{stp_{eu,t}}{RLTLREU_t \cdot RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bl_{eu,t}}{PLJ_{eu,t}} \cdot \left(YDISP_{eu,t} + PLJ_{eu,t} \cdot LJV_{eu,t} - OBL_{eu,t}\right) & if\ er = euc27 \\ cl_{eu,t} + \dfrac{stp_{eu,t}}{RLTLR_{er,t} \cdot RLTLRWORLD_t} \cdot \dfrac{bl_{eu,t}}{PLJ_{eu,t}} \cdot \left(YDISP_{eu,t} + PLJ_{eu,t} \cdot LJV_{eu,t} - OBL_{eu,t}\right) & if\ er \neq euc27 \end{cases}$$

$$OBL_{eu,t} = PCI_{eu,t} \cdot ch_{eu,t} + PLJ_{eu,t} \cdot cl_{eu,t}$$

with the obliged consumption being now modified as to include the value of the minimum leisure consumed.

At the second stage, total consumption is further decomposed into demand for specific consumption goods. For this allocation an integrated model of consumer demand for non-durables and durables, developed by Conrad and Schröder (1991) is implemented.

The rationale behind the distinction between durables and non-durables is that the households obtain utility from consuming a non-durable good or service and from using a durable good. So for the latter the consumer has to decide on the desired stock of the durable based not only on the relative purchase cost of the durable, but also on the cost of those goods that are needed in connection with the durable (as for example fuels for cars or for heating systems).

The consumer problem can be written as:

$$\max U_C = \prod_{nd} (HCFV_i - chcfv_i)^{bhcfv_i} \prod_{dg} \left(SHINV_j^{fix} - chcfv_J \right)^{bhcfv_j} \qquad [8]$$

under the constraint:

$$\sum_{ND} PHCFV_i \cdot HCFV_i = PHCFV \cdot HCNDTOTV \qquad [9]$$

where:

U_c: the level of utility,

ND: index of non-durable goods,

DG: index of durable goods,

HCFV: consumption (in volume),

PHCFV: consumption price,

SHINV: stock of durables (assumed to be fixed),

chcfv: the obliged consumption in volume,

bhcfv: the share parameter per consumption category,

Non-durable goods and services are denoted by the index ND while durables by the index DG.

The Lagrangian of the problem is:

$$\mathcal{L} = \prod_{nd} (HCFV_i - chcfv)^{bhcfv_i} \prod_{dg} \left(SHINV_j^{fix} - chcfv_j \right)^{bhcfv_j} - \lambda \cdot \left(\sum_{nd} PHCFV_i \cdot HCFV_i - PHCFV \cdot HCNDTOTV \right)$$

Taking the first order conditions:

$$\frac{\partial \ell}{\partial HCFV_i} = \frac{BHCFV_i}{HCFV_i - chcfv_i} \cdot \prod_{nd} (HCFV_i - chcfv_i)^{bhcfv_i} \prod_{dg} \left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j} - \lambda \cdot PHCFV_i \equiv 0$$

$$\Rightarrow HCFV_i - chcfv_i = \frac{bhcfv_i}{PHCFV_i} \prod_{nd} (HCFV_i - chcfv_i)^{bhcfv_i} \prod_{dg} \left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j} \cdot \frac{1}{\lambda}$$

Substituting the above equation into :

$$\sum_{ND} PHCFV_i \cdot HCFV_i = PC \cdot HCNDTOTV \Rightarrow \sum_{nd} PHCFV_i \cdot (HCFV_i - chcfv_i)$$

$$= PHCFV \cdot HCNDTOTV - \sum_{nd} PHCFV_i \cdot chcfv_i$$

$$\Rightarrow \sum_{nd} PHCFV_i \cdot \left(\frac{bhcfv_i}{PC_i} \prod_{nd} (HCFV_i - chcfv_i)^{bhcfv_i} \cdot \prod_{dg} \left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j} \cdot \frac{1}{\lambda} \right)$$

$$= PHCFV \cdot HCNDTOTV - \sum_{nd} PHCFV_i \cdot CHCFV_i \Rightarrow \frac{1}{\lambda}$$

$$= \frac{1}{\sum_{nd} PHCFV_i \cdot \frac{bhcfv_i}{PHCFV_i} \prod_{nd} (HCFV_i - chcfv_i)^{bhcfv_i} \cdot \prod_{dg} \left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j}}$$

$$\cdot \left(PHCFV \cdot HCNDTOTV - \sum_{nd} PHCFV_i \cdot chcfv_i \right)$$

Using the value of the multiplier, the demand functions to be used in the model are obtained:

$$HCFV_{nd,er,t} = chcfv_{nd,er,t} + \left(\frac{bhcfv_{nd,er,t}}{PHCFV_{nd,er,t}} \right) \cdot \left(HCNDTOT_{er,t} - \sum_{nd} PHCFV_{nd,er,t} \cdot chcfv_{nd,er,t} \right) \qquad [10]$$

By substituting demand function [10] in the utility function [8], one can derive the following expenditure function for non-durables (Schröder (1991) :

$$U_C = \prod_{nd}(HCFV_i - chcfv_i)^{bhcfv_i}\prod_{DG}\left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j} \Rightarrow U_C$$
$$= \prod_{nd}\left\{\frac{bhcfv_i}{PHCFV_i}\cdot\left(HCNDTOT - \sum_{ND} PHCFV_i \cdot chcfv_i\right)\right\}^{bhcfv_i}\prod_{DG}\left(SHINV_j^{fix} - chcfv_j\right)^{bhcfv_j}$$
$$\Rightarrow \prod_{nd}\left(HCNDTOT - \sum_{ND} PHCFV_i \cdot chcfv_i\right)^{bhcfv_j} = U_C$$
$$\cdot \prod_{dg}\left(SHINV_j^{fix} - chcfv_j\right)^{-bhcfv_j}\prod_{nd}\left(\frac{bhcfv_i}{PHCFV_i}\right)^{-bhcfv_i}$$
$$\Rightarrow \left(HCNDTOT - \sum_{nd} PHCFV_i \cdot chcfv_i\right)^{\sum_{nd} bhcfv_i}$$
$$= U_C \cdot \prod_{dg}\left(SHINV_j^{fix} - chcfv_j\right)^{-bhcfv_j}\prod_{nd}\left(\frac{bhcfv_i}{PHCFV_i}\right)^{-bhcfv_i} \Rightarrow HCNDTOT$$
$$= \sum_{nd} PHCFV_i \cdot chcfv_i + U_C \cdot \prod_{dg}\left(SHINV_j^{fix} - chcfv_j\right)^{-bhcfv_j}\prod_{nd}\left(\frac{bhcfv_i}{PHCFV_i}\right)^{-bhcfv_i}$$

Therefore:

$$E(U,p,sdg) = \sum_{nd} PHCFV_{nd} \cdot chcfv_{nd} + U_C \cdot \prod_{dg}\left(SHINV - chcfv_{dg}\right)^{-bhcfv_{dg}} \cdot \prod_{nd}\left(\frac{PHCFV_{nd}}{bhcfv_{nd}}\right)^{bhcfv_{nd}} \qquad [11]$$

where HCNDTOT is equal to E, the total expenditure on non -durables, which gives the (minimum) expenditure on non-durables given the stock of durables and the utility level U. By assuming that the household decides the amount of stock of durables the cost of using a durable is obtained by differentiating the above expenditure function with respect to the stock of each of the durables:

$$\frac{\partial E}{\partial SHINV} = -\frac{bhcfv_{dg} \cdot (HCNDTOT - \sum_{nd} PHCFV_{nd} \cdot chcfv_{nd})}{SHINV - chcfv_{dg}}$$

The cost of operating the durables (i.e. the consumption of linked non-durables) is included in the user's cost of the durable *PDUR*:

$$PDUR_{dg,er,t} = \begin{cases} PHCFV_{dg,er,t} \cdot \left(RLTLREU_t \cdot RLTLR_{er,t} \cdot RLTLRWORLD_t + declh_{dg,er,t}\right) + \\ + txproperty_{dg,er,t} \cdot \left(1 + RLTLREU_t \cdot RLTLR_{er,t} \cdot RLTLRWORLD_t\right) + \\ + \sum_{lnd}\left(\frac{dispcons_{lnd,dg,er,t} + mincons_{lnd,dg,er,t}}{efi\_llndc_{lnd,dg,er,t}} \cdot PUHCFVDG_{lnd,dg,er,t}\right) & if\ er = euc27 \\ PHCFV_{dg,er,t} \cdot \left(RLTLR_{er,t} \cdot RLTLRWORLD_t + declh_{dg,er,t}\right) + \\ + txproperty_{dg,er,t} \cdot \left(1 + RLTLR_{er,t} \cdot RLTLRWORLD_t\right) + \\ + \sum_{lnd}\left(\frac{dispcons_{lnd,dg,er,t} + mincons_{lnd,dg,er,t}}{efi\_llndc_{lnd,dg,er,t}} \cdot PUHCFVDG_{lnd,dg,er,t}\right) & if\ er \neq euc27 \end{cases} \qquad [12]$$

where:

$declh_{dg,er,t}$: the replacement rate for durable goods,

$txproperty_{dg,er,t}$: the property tax for the durables,
lnd: the set defining all linked non-durable goods,

$PUHCFVDG_{lnd,dg,er,t}$: the user cost of linked non-durables including the abatement cost,

$mincons_{lnd,dg,er,t}$: the minimum consumption of the non-durable that is needed for a positive service flow to be created,

$dispcons_{lnd,dg,er,t}$: measures the proportion of the consumption of the linked non-durable good that is used along with the durable so as to provide positive service flow. That is the consumption of non -durables per unit of durable (e.g. consumption of gasoline by a car):

$$dispcons_{lnd,dg,er,t} = alphdisp_{lnd,dg,er,t} \cdot \left(\frac{PCI_{er,t}}{PHCFVDG_{lnd,dg,er,t}}\right)^{etadisp_{lnd,dg,er,t}} \quad [13]$$

where:

$alphadisp_{lnd,dg,er,t}$: a ratio coefficient,

$etadisp_{lnd,nd,er,t}$: a price elasticity.

$PCI_{er,t}$: the price index of private consumption in year *t*

The last part of the user cost equation links some non-durable goods to the use of durables. Energy is the main linked non-durable good. Energy complements the use of durables in order for them to provide a positive service flow. Consumption of energy does not affect the expenditure of durables through the change in preferences but rather through the additional burden in the user cost. To calculate the desired stock levels of the durables, this quantity is set equal to the marginal cost of holding one more unit of durable goods for one period. The desired stock of the durables is:

$$SHINV_{fn,er,t} = chcfv_{fn,er,t} + \left(\frac{bhcfv_{fn,er,t}}{PDUR_{fn,er,t}}\right) \cdot \left(HCNDTOT_{er,t} - \sum_{nd}\left(PHCFV_{nd,er,t} \cdot chcfv_{nd,er,t}\right)\right) \quad [14]$$

where:

$PHCFV_{er,t}$: the price of private consumption category,

The demand for linked non-durable goods, coupled with the use of the durable is then:

$$LLNDC_{lnd,dg,er,t} = \frac{dispcons_{lnd,dg,er,t} + mincons_{lnd,dg,er,t}}{effi\_llndc_{lnd,dg,er,t}} \cdot SHINV_{LND,dg,er,t} \quad [15]$$

where:

$efi\_llndc_{lnd,dg,er,t}$: efficiency parameter for household.

If there is no need for the use of the non-durable good, $mincons_{lnd,dg,er,t}$ in the first equation of the linked non-durables becomes zero, and thus:

$$HCFV_{fn,er,t} = chcfv_{nd,er,t} + \frac{bhcfv_{fn,er,t}}{PHCFV_{fn,er,t}}\left[HCNDTOT_{er,t} - \sum_{nd} PHCFV_{fn,er,t} \cdot chcfv_{fn,er,t}\right] + \sum_{dg} LLNDC_{fn,dg,er,t} \qquad [16]$$

Total households' expenditure is then the sum of consumption (for non-linked non-durables) plus investment in durables plus consumption in non-durables used with durables.

$$HCDTOT_{er,t} = HCNDTOT_{er,t} + \sum_{dg}\left(HCFV_{dg,er,t} \cdot PHCFV_{dg,er,t}\right) + \sum_{lnd}\left(\sum_{dg} LLNDC_{lnd,dg,er,t} \cdot PHCFV_{dg,er,t}\right) \qquad [17]$$

where:

$\sum_{dg} HCFV_{dg,er,t}$: represents the change in stocks of durables or in other words, the net investment that is necessary to move towards the long run equilibrium durable goods levels.

Assuming a rate of replacement *declh*, this investment is equal to:

$$HCFV_{dg,er,t} = \left(\frac{SHINV_{dg,er,t} - \left(1 - declh_{dg,er,t}\right)^{PERIOD} \cdot SHINV_{dg,er,t-1}}{\left(1 - declh_{dg,er,t}\right)^{PERIOD} - 1}\right) \cdot \left[\left(1 - declh_{dg,er,t}\right) - 1\right] \qquad [18]$$

where $SHINV_{dg,er,t-1}$ is the stock of durable goods of the previous period, which is known in the current period. The demand for consumption categories is then transformed into demand for products through a consumption transition matrix with fixed technical coefficients:

$$HCV_{pr,er,t} = \sum_{fn}\left(\frac{thcfv_{pr,fn,er,t}}{e^{tgqtch_{pr,fn,er,t}}} \cdot HCFV_{pr,fn,er,t}\right) \qquad [19]$$

Equation [19] determines the final consumption expenditure of the households. The total consumption, for all goods, in a country is given by:

$$HCDTOTV_{er,t} = \sum_{pr} HCV_{pr,er,t} \qquad [20]$$

The consumption transition matrix is also used to compute the consumption price by function, as the weighted average of the delivery prices of products to private consumption (PH):

$$PHCFV_{fn,er,t} = \begin{cases} \sum_{pr}\left(\frac{thcfv_{pr,fn,er,t}}{e^{tgqtch_{pr,fn,er,t}}} \cdot PHC_{pr,er,t}\right) & if\ fn = nlnd \\ \frac{\sum_{dg} PUHCFVDG_{fn,DG,er,t} \cdot LLNDC_{fn,dg,er,t}}{\sum_{dg} LLNDC_{fn,dg,er,t}} & if\ fn = lnd \end{cases} \qquad [21]$$

A cost-of-living index can be derived as the ratio between the value and the volume of consumption; it gives the change in the consumer price relative to the numeraire.

$$PCI_{er,t} = \frac{\sum_{pr}(PHC_{pr,er,t} \cdot HCV_{pr,er,t})}{\sum_{pr} HCV_{pr,er,t}} \qquad [22]$$

4.2 Firms behaviour

Each producer (represented by an activity) is assumed to maximize profits, defined as the difference between the revenue earned and the cost of factors and intermediate inputs. Profits are maximized subject to its production technology. Domestic production is defined by branch. It is assumed that each branch produces a single good which is differentiated from any other good in the economy. Production functions in GEM-E3 exhibit a nested separability scheme, involving capital (K), skilled and unskilled labour (L_{skld}, L_{unskld}), energy (E) and materials (M) and are based on a CES neo-classical type of production function. The exact nesting scheme of production in GEM-E3 has been selected to match available econometric data on KLEM substitution elasticities and the specific features of each activity. The optimal production behaviour can be represented in the primal or the dual formulation. Their equivalence, under certain assumptions, can be verified by the theory of production behaviour and is illustrated with the following formulations.

The primal formulation is given by:

$$XD_i = \sum_j \left[\delta_{i,j}^{\frac{1}{\sigma}} \cdot X_{i,j}^{\frac{\sigma-1}{\sigma}} \cdot e^{(\sigma-1)\cdot tpj\cdot t} \right]^{\frac{\sigma}{\sigma-1}}$$

$$X_{i,j} = XD_i \cdot \delta_{i,j} \cdot \left(\frac{P_i \cdot e^{(\sigma-1)\cdot tpj\cdot t}}{PX_{i,j}} \right)^{\sigma}$$

$P_i \cdot XD_i = \sum_j PX_{i,j} \cdot X_{i,j}$ (zero profit condition)

where:

XD_i: production in volume,

$X_{i,j}$: production factor,

P_i: the output price of domestic production,

$\delta_{i,j}$: scale factors for the production factors (intermediate consumption, energy, capital and labour),

$PX_{i,j}$: the price of the factor j,

σ: the elasticity of substitution.

The last factor in the equation reflects the technical progress that is embedded in the production factors (*tpj* is the rate of technical progress embedded in production factor *j*).

The dual formulation is given by:

$$P_i = \sum_j \left[\delta_{i,j} \cdot PX_{i,j}^{1-\sigma} \cdot e^{(\sigma-1)\cdot tpj\cdot t} \right]^{\frac{1}{1-\sigma}}$$

$$X_{i,j} = XD_i \cdot \delta_{i,j} \cdot \left(\frac{P_i \cdot e^{(\sigma-1)\cdot tpj\cdot t}}{PX_{i,j}}\right)^{\sigma}$$

$P_i \cdot XD_i = \sum_j PX_{i,j} \cdot X_{i,j}$ (zero profit condition)

It can be proved, that under constant returns to scale, the two formulations are exactly the same. In both formulations, an equation for the equality between desired and existing capital is added and one of the *(j+1)* equations (*j* derived demand functions and the zero profit condition) are redundant:

- Either the demand of capital is redundant and the zero profit condition serves to compute the rate of return on capital.
- Or the zero profit equation is suppressed and the equilibrium on the capital market determines the rate of return on capital.

It is easy to prove that the primal and the dual formulation to the same solution.
In the model the dual formulation is used and the long run unit cost function is of the nested CES type with factor-augmenting technical change, i.e. price diminishing technical change. The firm (at branch level) decides its supply of goods or services given its selling price and the prices of production factors. The production technology exhibits constant return of scale. The firm supplies its good and selects a production technology so as to maximise its profit within the current year, given the fact that the firm cannot change the stock of productive capital within this period of time. The firm can change its stock of capital the following year, by investing in the current one. Since the stock of capital is fixed within the current year, the supply curve of domestic goods is upwards sloping and exhibits decreasing return to scale[13].

Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 represent the nesting structure for the different activities included in the GEM-E3 model.

Non-energy sectors: At the 1st level, production is split into two aggregates, one consisting of capital, labour, energy bundle (*KLE*) and the other consisting of materials (*MA*). At the 2nd level, (*KLE*) is split in two aggregates, one consisting of capital and labour bundle (*KL*), and the other consisting of energy (*ENG*). (*MA*) is further divided in its component parts (e.g. Agriculture, Industrial activities, Services etc.). At the 3rd level (*KL*) is split into capital and skilled labour bundle (KL_skld), which is further decomposed at the 4th level between Capital and skilled Labour, and unskilled labour (*L_unskld*), whereas (*ENG*) is split in electricity and fuels (*EN*) (Figure 4).

Resource sectors: For the sectors whose production is based on natural resources, at the 1st nesting level production is split between fossil fuel resources (*RES*) and an aggregate bundle consisting of capital, labour and material-energy (*KLEMrs*). The latter at the 2nd stage is disaggregated in the material-energy bundle (*MAENrs*) and the capital-labour bundle (*KL).* At the 3rd level the capital-labour bundle (*KL*) is split in capital and skilled labour (KL_skld) and in unskilled labour. The material-energy bundle (*MAENrs*) is divided

[13] This description applies only to the most rigid of the capital mobility assumptions that are available in the model variants, where capital is assumed immobile across sectors and countries in static terms. When capital is assumed malleable across sectors and/or countries, then the capital stock by sector can adjust even in static terms, but the overall capital resources available to the economy (of the country or the EU as a whole) within each time-period are constant.

into its component parts. Finally capital-skilled labour bundle is spit into capital and skilled labour (Figure 5).

Power supply sectors: At the 1st nesting level of the power supply sector, production is split into two aggregates, one consisting of a bundle of power producing technologies (*TECH*) and the other of the transmission and distribution part (*DIST*). At the 2nd level, all power producing technologies identified in the model are in the same nest whereas the (*DIST*) bundle is disaggregated to capital, skilled and unskilled labour and materials (Figure 6).

Power producing technologies: one level production function that includes capital, skilled and unskilled labour and fuels is assumed (Figure 7).

Refineries: the nesting structure is similar to the non-energy sectors with a change in the top level of the nest where the two aggregates are now (*KLEM*) and fuels (*FUEL*) (Figure 8).

Figure 4: Production nesting scheme in the GEM-E3 model – Non energy sectors

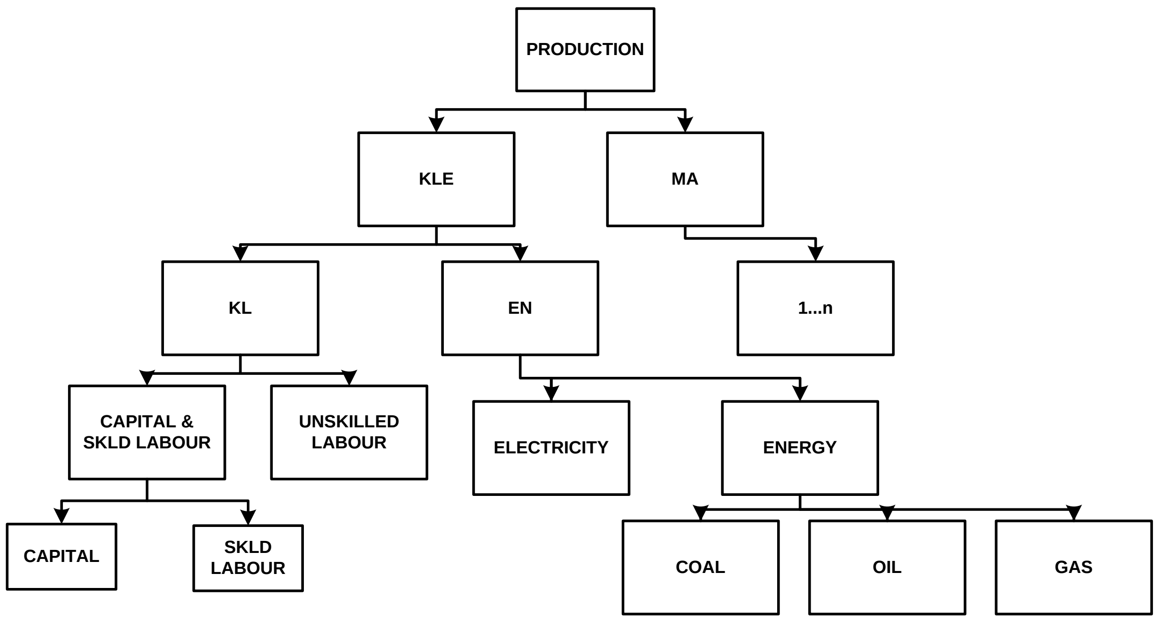


Figure 5: Production nesting scheme in the GEM-E3 model – Resource sectors

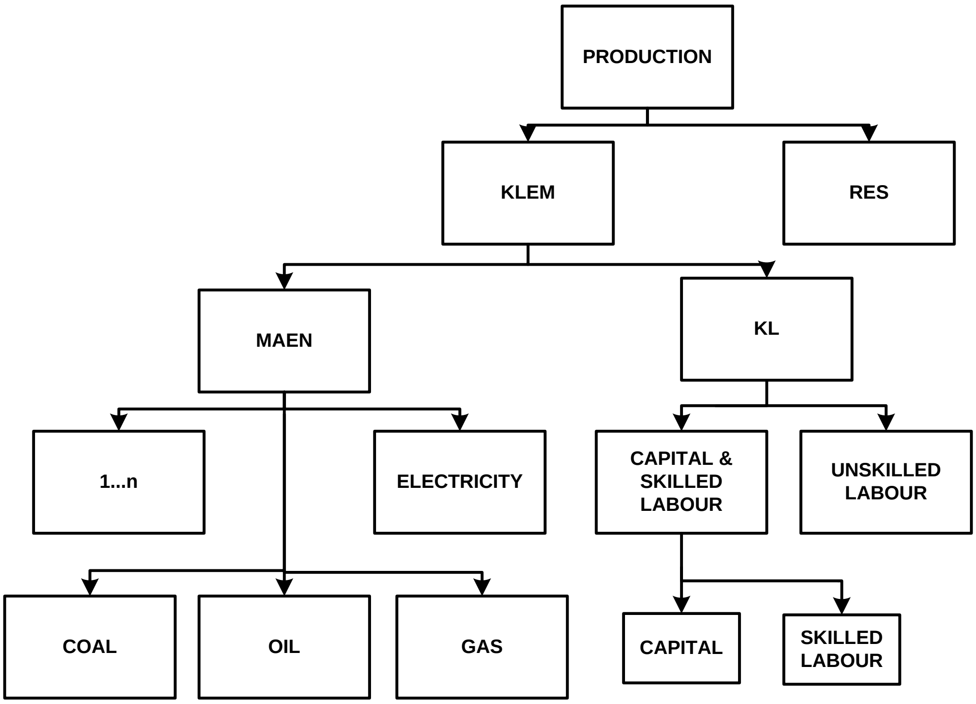


Figure 6: Production nesting scheme in the GEM-E3 model – Electricity supply

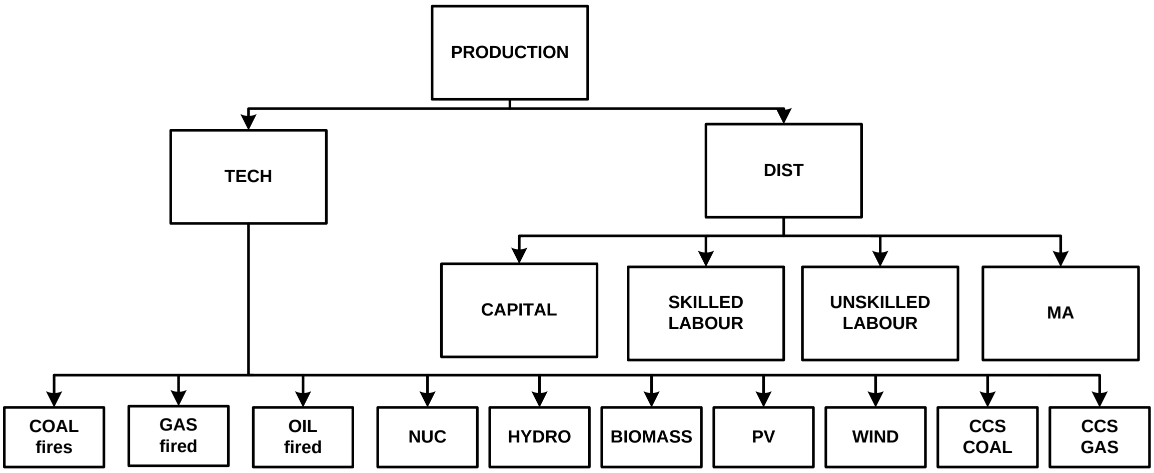


Figure 7: Production nesting scheme in the GEM-E3 model – Power producing technologies

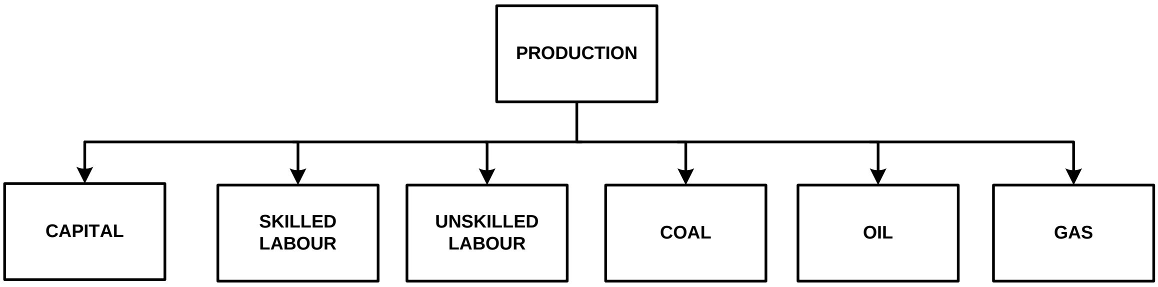
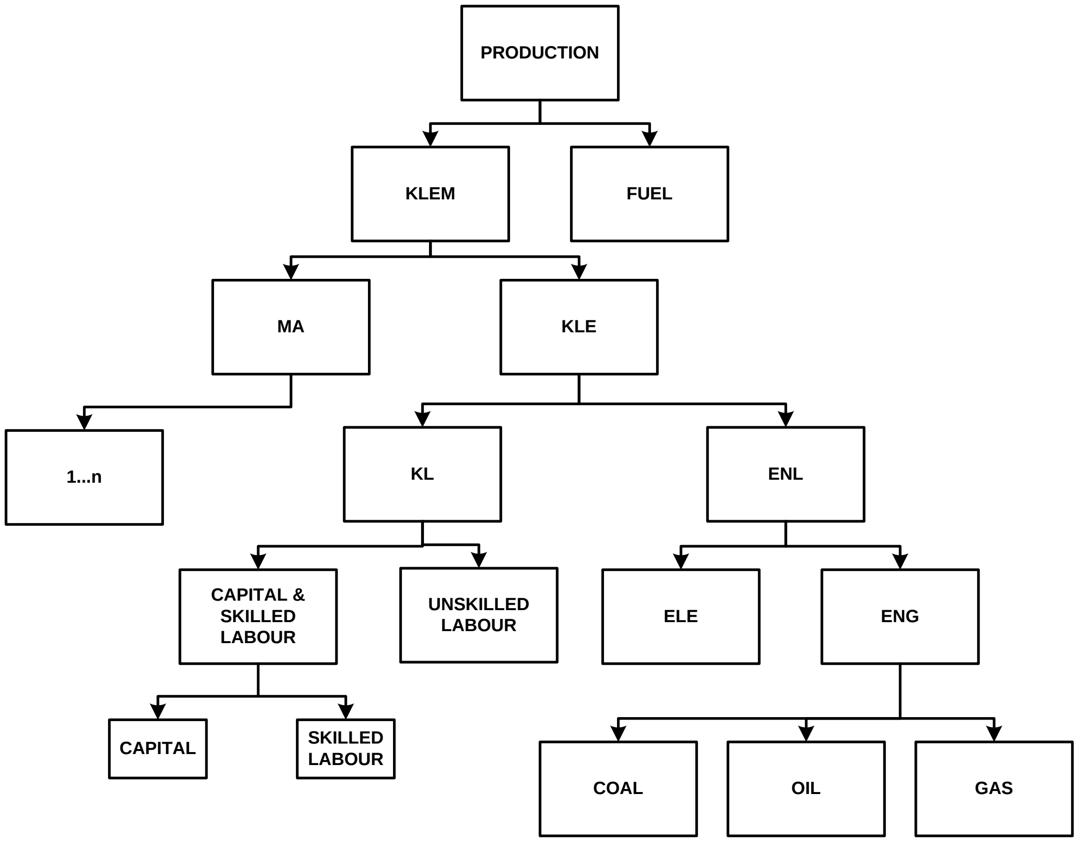


Figure 8: Production nesting scheme in the GEM-E3 model – Refineries



Below we provide the derivation of the optimal factor demands and the unit cost function for a two factor production function. The 1st nest of the production function has the following form (consider the case of the non-energy sectors):

$$XD_{pr} = [tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}] \cdot XD0_{pr} \left[theta\_\delta_{KLE,pr}^{\frac{1}{\sigma_1}} \cdot \left(\frac{KLE_{pr}}{KLE0_{pr}}\right)^{\frac{\sigma_1-1}{\sigma_1}} + theta\_\delta_{MA,pr}^{\frac{1}{\sigma_1}} \cdot \right]^{\frac{\sigma_1}{\sigma_1-1}}$$

where:

XD_{pr}: the domestic production,

KLE_{pr}: the Capital-Labour-Electricity bundle,

MA_{pr}: the Materials bundle in production,

σ_1: the elasticity of substitution between KLE_{pr} and MA_{pr},

tfp: the total factor productivity,

tfpexo: the exogenous total factor productivity,
$\mathrm{theta\_\delta_{KLE,pr}}$ $theta\_\delta_{KLE,pr}$ and $theta_\delta_{MA,pr}$$\mathrm{theta_\delta_{MA,pr}}$: value shares derived from the base year dataset.

These value shares are calibrated using the observed values and volumes in the base year:

$$theta\_\delta_{KLE,pr} = \frac{pKLE0_{pr} \cdot KLE0_{pr}}{pD0_{pr} \cdot XD0_{PR}}$$

and

$$theta\_\delta_{MA,pr} = \frac{pM0_{pr} \cdot MA0_{pr}}{pD0_{pr} \cdot XD0_{PR}}$$

The dual function representing the unit production cost, on the other hand, is expressed in the following way:

$$PD_{pr,er,t} = \frac{pD0_{pr,er}}{tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}} \cdot \left[theta\_\delta_{KLE,pr,er,t} \cdot \left(\frac{PKLE_{pr,er,t}}{PKLE0_{pr,er}} \right)^{1-\sigma_{1_{pr,er,t}}} + theta\_\delta_{MA,pr,er,t} \cdot \left(\frac{PM_{pr,er,t}}{PM0_{pr,er}} \right)^{1-\sigma_{1_{pr,er,t}}} \right]^{\frac{1}{1-\sigma_{1_{pr,er,t}}}}$$

where:

$PD_{pr,er,t}$: the deflator in domestic production,

$PKLE_{pr,er,t}$: the deflator of Capital-Labour-Electricity bundle,

$PM_{pr,er,t}$: the deflator of Materials bundle.

Optimal factor demand is derived from Shephard's lemma. The assumption that the stocks of capital and labour are proportional to the optimal flows (i.e. the capital and labour services derived through the Shephard's lemma) in volume is made.

In particular the cost minimization problem (for the 1st nest) is:

$$\min C_{pr,er,t} = PKLE_{pr,er,t} \cdot KLE_{pr,er,t} + PM_{pr,er,t} \cdot MA_{pr,er,t}$$

such that:

$$\overline{XD}_{pr,er,t} = \left[theta\_\delta_{KLE,pr,er,t}^{1/\sigma_{1_{pr,er,t}}} \cdot \frac{KLE_{pr,er,t}}{KLE0_{pr,er}}^{\frac{\sigma_{1_{pr,er,t}}-1}{\sigma_{1_{pr,er,t}}}} + theta\_\delta_{MA,pr,er,t}^{1/\sigma_{pr,er,t}} \cdot \frac{MA_{pr,er,t}}{MA0_{pr,er}}^{\frac{\sigma_{1_{pr,er,t}}-1}{\sigma_{1_{pr,er,t}}}} \right]^{\frac{\sigma_{1_{pr,er,t}}}{\sigma_{1_{pr,er,t}}-1}}$$

Solving the cost minimization problem and using Shephard’s lemma we obtain the following compensated demand function:

$$MA_{pr,er,t} = MA0_{pr,er} \cdot \overline{XD}_{pr,er,t} \cdot theta\_\delta_{MA,pr,er,t} \cdot \left(\frac{PM0_{pr,er}}{PM_{pr,er,t}} \cdot \frac{C_{pr,er,t}}{C0_{pr,er,t}} \right)^{\sigma_{1pr,er,t}}$$

$$KLE_{pr,er,t} = KLE0_{pr,er} \cdot \overline{XD}_{pr,er,t} \cdot theta\_\delta_{KLE,pr,er,t} \cdot \left(\frac{PKLE0_{pr,er}}{PKLE_{pr,er,t}} \cdot \frac{C_{pr,er,t}}{C0_{pr,er,t}} \right)^{\sigma_{1pr,er,t}}$$

Where $C0_{pr,er,t}$ is the cost function at the benchmark year.

$$C0_{pr,er,t} = PKLE0_{pr,er} \cdot KLE0_{pr,er} + PM0_{pr,er} \cdot MA0_{pr,er}$$

Similar results are obtained when technological progress or factor productivities are included in one or both factors:

$$MA_{pr,er,t} = MA0_{pr,er} \cdot \overline{XD}_{pr,er,t} \cdot theta\_\delta_{MA,pr,er,t} \cdot \left(\frac{PM0_{pr,er}}{PM_{pr,er,t}} \cdot \frac{C_{pr,er,t}}{C0_{pr,er,t}}\right)^{\sigma_{1pr,er,t}} \cdot [tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}]^{\left(\sigma_{1pr,er,t}-1\right)}$$

$$KLE_{pr,er,t} = KLE0_{pr,er} \cdot \overline{XD}_{pr,er,t} \cdot theta\_\delta_{KLE,pr,er,t} \cdot \left(\frac{PKLE0_{pr,er}}{PKLE_{pr,er,t}} \cdot \frac{C_{pr,er,t}}{C0_{pr,er,t}}\right)^{\sigma_{1pr,er,t}} \cdot [tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}]^{\left(\sigma_{1pr,er,t}-1\right)}$$

Similar formulas can be derived for each level of the nesting scheme of the production function, always linking the demand for a factor at a lower level of the nesting scheme to the bundle to which it belongs, with different substitution elasticities at each level. This gives finally a cost-minimising demand for each production factor:

$$KAV_{prrs,er,t} = \left(\frac{KAV0_{pr,er} \cdot e^{tgk_{pr,er,t} \cdot (sn4_{pr,er,t}-1)} \cdot \frac{KLRS_{pr,er,t}}{KLRS0_{pr,er}}}{\left(\frac{PK_{pr,er,t}}{PKLRS_{pr,er,t}} \cdot \frac{PKLRS0_{pr,er}}{PK0_{pr,er}}\right)^{sn4}}\right) \quad [23]$$

$$KAV_{ele,er,t} = \frac{KAV0_{pr,er} \cdot e^{-tgk_{pr,er,t}} \cdot DIST_{pr,er,t}}{DIST0_{pr,er}} \quad [24]$$

$$KAV_{tec,er,t} = \frac{theta\_dkav_{pr,er,t} \cdot e^{-tgk_{pr,er,t}} \cdot XD_{pr,er,t}}{\left(tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}\right)^{-1}} \quad [25]$$

$$KAV_{pr,er,t} = \left(\frac{KAV0_{pr,er} \cdot e^{tgk_{pr,er,t} \cdot (sn4_{pr,er,t}-1)} \cdot \frac{KL_{\substack{pr,er,t\\skld}}}{KL0_{\substack{pr,er\\skld}}}}{\left(\frac{PK_{pr,er,t}}{PKL_{\substack{pr,er,t\\skld}}} \cdot \frac{PKL0_{\substack{pr,er\\skld}}}{PK0_{pr,er}}\right)^{sn4_{pr,er,t}}}\right) \quad [26]$$

$$LAV_{\substack{prrs,er,t\\skld}} = LAV0_{\substack{prrs,er\\skld}} e^{tgl\_skld(sn4-1)} \frac{KLRS_{\substack{pr,er\\skld}}}{KLRS0_{\substack{pr,er\\skld}}} \left(\frac{PKLRS_{\substack{pr,er,t\\skld}}}{PL_{\substack{pr,er,t\\skld}}} \frac{PL0_{\substack{pr,er\\skld}}}{PKLRS0_{\substack{pr,er\\skld}}}\right)^{sn4} \quad [27]$$

$$\boldsymbol{LAV_{\substack{prrs,er,t\\unskld}} = LAV0_{\substack{prrs,er\\unskld}} e^{tgl\_unskld(sn4-1)} \frac{KLRS_{\substack{pr,er,t\\unskld}}}{KLRS0_{\substack{pr,er\\unskld}}} \left(\frac{PKLRS_{\substack{pr,er,t\\unskld}}}{PL_{\substack{pr,er,t\\unskld}}} \frac{PL0_{\substack{pr,er\\unskld}}}{PKLRS0_{\substack{pr,er\\unskld}}}\right)^{sn4}} \quad [28]$$

$$LAV_{\substack{ele,er,t\\skld}} = \frac{LAV0_{\substack{pr,er\\skld}} \cdot e^{-tgl\_skld_{pr,er,t}} \cdot DIST_{pr,er,t}}{DIST0_{pr,er}} \quad [29]$$

$$LAV_{\substack{ele,er,t\\unskld}} = \frac{LAV0_{\substack{pr,er\\unskld}} \cdot e^{-tgl\_unskld_{pr,er,t}} \cdot DIST_{pr,er,t}}{DIST0_{pr,er,t}} \quad [30]$$

$$LAV_{\substack{tec,er,t\\skld}} = \frac{dl_{\substack{pr,er,t\\skld}} \cdot \frac{PDBSR0_{pr,er}}{PDBSR_{pr,er,t}} \cdot e^{-tgl\_skld_{pr,er,t}} \cdot XD_{pr,er,t}}{\left(tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}\right)^{-1}} \quad [31]$$

$$LAV_{\substack{tec,er,t\\unskld}} = \frac{dl_{\substack{pr,er,t\\unskld}} \cdot \frac{PDBSR0_{pr,er}}{PDBSR_{pr,er,t}} \cdot e^{-tgl\_unskld_{pr,er,t}} \cdot XD_{pr,er,t}}{\left(tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}\right)^{-1}} \quad [32]$$

$$LAV_{\underset{skld}{pr,er,t}} = \left(\frac{LAV0_{\underset{skld}{pr,er}} \cdot e^{tgl\_skld_{pr,er,t} \cdot (sn4_{pr,er,t}-1)} \cdot \frac{KL_{\underset{skld}{pr,er,t}}}{KL0_{\underset{skld}{pr,er}}}}{\left(\frac{PL_{\underset{skld}{pr,er,t}} \cdot PKL0_{\underset{skld}{pr,er}}}{PKL_{\underset{skld}{pr,er,t}} \cdot PL0_{\underset{skld}{pr,er}}} \right)^{sn4_{pr,er,t}}} \right) \quad [33]$$

$$LAV_{\underset{unskld}{pr,er,t}} = \left(\frac{LAV0_{\underset{unskld}{pr,er}} \cdot e^{tgl\_unskld_{pr,er,t} \cdot (sn4_{pr,er,t}-1)} \cdot \frac{KL_{\underset{skld}{pr,er,t}}}{KL0_{\underset{skld}{pr,er}}}}{\left(\frac{PL_{\underset{skld}{pr,er,t}} \cdot PKL0_{\underset{skld}{pr,er}}}{PKL_{\underset{skld}{pr,er,t}} \cdot PL0_{\underset{skld}{pr,er}}} \right)^{sn4_{pr,er,t}}} \right) \quad [34]$$

$$IOV_{pr,prdf,er,t} = \begin{cases} MA_{br,er,t} \cdot \frac{PM0_{br,er}}{PIO0_{pr,er}} \cdot theta\_dmpr_{pr,br,er,t} \cdot \left(\frac{PM_{br,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{br,er}}{PM0_{pr,er}} \right)^{sn3_{br,er,t}} \left[e^{tgm_{pr,br,er,t}(sn3_{br,er,t}-1)} \right] & if\ pr \\ EN_{br,er,t} \cdot \frac{PE_{0br,er}}{PIO0_{pr,er}} \cdot theta\_depr_{pr,br,er,t} \cdot \left(\frac{PE_{br,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{br,er}}{PE0_{pr,er}} \right)^{sn6_{br,er,t}} \left[e^{tge_{pr,br,er,t}(sn6_{br,er,t}-1)} \right] & if\ pr \\ ENL_{br,er,t} \cdot \frac{IOV0_{pr,br,er}}{ENL0_{br,er}} & if\ pr \end{cases} \quad [35]$$

$$IOV_{pr,proil,er,t} = \begin{cases} XD_{pr,er,t} \cdot \frac{PDBSR0_{br,er}}{PIO0_{pr,er}} \cdot theta\_dio_{pr,br,er,t} \cdot \left(\frac{PDBSR_{br,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{pr,er}}{PDBRS0_{br,er}} \right)^{sn0_{br,er,t}} \cdot \left[tfp_{br,er,t} \cdot tfpexo_{br,er,t} \right]^{sn0_{br,er,t}-1} & if\ pr \\ MA_{br,er,t} \cdot \frac{PM0_{br,er}}{PIO0_{pr,er}} \cdot theta\_dmpr_{pr,br,er,t} \cdot \left(\frac{PM_{br,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{pr,er}}{PM0_{br,er}} \right)^{sn3_{br,er,t}} \left[e^{tgm_{pr,br,er,t}(sn3_{br,er,t}-1)} \right] & if\ pr \\ EN_{br,er,t} \cdot \frac{PE0_{br,er}}{PIO0_{pr,er}} \cdot theta\_depr_{pr,br,er,t} \cdot \left(\frac{PE_{br,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIOO_{pr,er}}{PE0_{br,er}} \right)^{sn6_{br,er,t}} \left[e^{tge_{pr,br,er,t}(sn6_{br,er,t}-1)} \right] & if\ pr \\ ENL_{br,er,t} \cdot \frac{IOV0_{pr,br,er}}{ENL0_{br,er}} & if\ pr \end{cases} \quad [36]$$

$$IOV_{pr,tec,er,t} = \frac{theta\_dio_{pr,br,er,t} \cdot \frac{PDBSR0_{br,er}}{PIO0_{pr,er}} \cdot e^{-tgio_{pr,br,er,t}} \cdot XD_{pr,er,t}}{tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}} \quad [37]$$

$$IOV_{pr,ele,er,t} = \begin{cases} IOV0_{pr,br,er} \cdot e^{-tgm_{pr,br,er,t}} \cdot \frac{DIST_{br,er,t}}{DIST0_{br,er}} & if\ pr = prane \\ IOV0_{pr,er} \cdot e^{-tgm_{pr,br,er,t}} \cdot \frac{DIST_{br,er,t}}{DIST0_{br,er}} & if\ pr = pre, ele \end{cases} \quad [38]$$

$$IOV_{pr,prrs,er,t} = \begin{cases} MA_{prrs,er,t} \cdot \frac{PM0_{prrs,er}}{PIO0_{pr,er}} \cdot theta\_dmpr_{pr,prrs,er,t} \cdot \left(\frac{PM_{prrs,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{pr,er}}{PM0_{prrs,er}} \right)^{snrs2_{prrs,er,t}} \left[e^{tgm_{pr,prrs,er,t}(sn3_{prrs,er,t}-1)} \right] & if\ pr = \\ EN_{prrs,er,t} \cdot \frac{PE0_{prrs,er}}{PIO0_{pr,er}} \cdot theta\_dprrs_{pr,prrs,er,t} \cdot \left(\frac{PE_{prrs,er,t}}{PIO_{pr,er,t}} \cdot \frac{PIO0_{pr,er}}{PE0_{prrs,er}} \right)^{snrs2_{prrs,er,t}} \left[e^{tge_{pr,prrs,er,t}(sn6_{prrs,er,t}-1)} \right] & if\ pr = \\ IOV0_{prrs,br,er} \cdot \frac{ENL_{prrs,er,t,}}{ENL00_{pr,er}} & if\ pr = \end{cases} \quad [39]$$

$$KLE_{pr,er,t} = \begin{cases} XD_{pr,er,t} \cdot \frac{PDBSR0_{pr,er}}{PKLE0_{pr,er}} \cdot theta\_dkle_{pr,er,t} \cdot \left(\frac{PDBSR_{pr,er,t}}{PKLE_{pr,er,t}} \cdot \frac{PKLE0_{pr,er}}{PDBSR0_{pr,er}} \right)^{sn1_{pr,er,t}} \left[tfp_{pr,er,t} \cdot tfpexo_{pr,er,t} \right. \\ KLEM_{pr,er,t} \cdot \frac{PKLEM0_{pr,er}}{PKLE0_{pr,er}} \cdot theta\_dkle_{pr,er,t} \cdot \left(\frac{PKLEM_{pr,er,t}}{PKLE_{pr,er,t}} \cdot \frac{PKLE0_{pr,er}}{PKLEM0_{pr,er}} \right)^{sn1_{pr,er,t}} \end{cases} \quad [40]$$

$$MA_{pr,er,t} = \begin{cases} theta\_dm_{pr,er,t} \cdot \frac{PDBSR0_{pr,er}}{PM0_{pr,er}} \cdot XD_{pr,er,t} \cdot \left(\frac{PDBSR_{pr,er,t}}{PM_{pr,er,t}} \cdot \frac{PM0_{pr,er}}{PDBSR0_{pr,er}}\right)^{sn1_{pr,er,t}} \left[tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}\right]^{s} \\ theta\_dm_{pr,er,t} \cdot \frac{PMAEN0_{pr,er}}{PM0_{pr,er}} \cdot MAEN_{pr,er,t} \cdot \left(\frac{PMAEN_{pr,er,t}}{PM_{pr,er,t}} \cdot \frac{PM0_{pr,er}}{PMAEN0_{pr,er}}\right)^{snrs2_{pr,er,t}} \quad if\ p \\ theta\_dm_{pr,er,t} \cdot \frac{PKLEM0_{pr,er}}{PM0_{pr,er}} \cdot KLEM_{pr,er,t} \cdot \left(\frac{PKLEM_{pr,er,t}}{PM_{pr,er,t}} \cdot \frac{PM0_{pr,er}}{PKLEM0_{pr,er}}\right)^{snrs2_{pr,er,t}} \quad if\ pr \end{cases} \quad [41]$$

$$KL_{pr,er,t} = KLE_{pr,er,t} \cdot \frac{PKLE0_{pr,er}}{PKL0_{pr,er}} \cdot theta\_dkl_{pr,er,t} \cdot \left(\frac{PKLE_{pr,er,t}}{PKL_{pr,er,t}} \cdot \frac{PKL0_{pr,er}}{PKLE0_{pr,er}}\right)^{sn2_{pr,er,t}} \quad if\ pr = prdf\ or\ proil \quad [42]$$

$$ENG_{pr,er,t} = KLE_{pr,er,t} \cdot \frac{PKLE0_{pr,er}}{PENG0_{pr,er}} \cdot theta\_deng_{pr,er,t} \cdot \left(\frac{PKLE_{pr,er,t}}{PENG_{pr,er,t}} \cdot \frac{PENG0_{pr,er}}{PKLE0_{pr,er}}\right)^{sn2_{pr,er,t}} \quad [43]$$

$$ENL_{pr,er,t} = \begin{cases} ENG_{pr,er,t} \cdot \frac{PENG0_{pr,er}}{PEL0_{pr,er}} \cdot theta\_del_{pr,er,t} \cdot \left(\frac{PENG_{pr,er,t}}{PEL_{pr,er,t}} \cdot \frac{PEL0_{pr,er}}{PENG0_{pr,er}}\right)^{sn5_{pr,er,t}} \cdot \left[e^{\left(\sum_{ele} tge_{ele,pr,er,t}\right)\left(sn5_{pr,er,t}-1\right.}\right. \\ MAEN_{pr,er,t} \cdot \frac{PMAEN0_{pr,er}}{PEL0_{pr,er}} \cdot theta\_del_{pr,er,t} \cdot \left(\frac{PMAEN_{pr,er,t}}{PEL_{pr,er,t}} \cdot \frac{PEL0_{pr,er}}{PMAEN0_{pr,er}}\right)^{snrs2_{pr,e}} \\ \cdot \left[e^{\left(\sum_{ele} tge_{ele,pr,er,t}\right)\left(snrs2_{pr,er,t}-1\right)}\right] \ if\ pr = prrs \end{cases} \quad [44]$$

$$EN_{pr,er,t} = \begin{cases} ENG_{pr,er,t} \cdot \frac{PENG0_{pr,er}}{PE0_{pr,er}} \cdot theta\_de_{pr,er,t} \cdot \left(\frac{PENG_{pr,er,t}}{PE_{pr,er,t}} \cdot \frac{PE0_{pr,er}}{PENG0_{pr,er}}\right)^{sn5_{pr,er,t}} \quad \text{if pr = prdf or proil} \\ MAEN_{pr,er,t} \cdot \frac{PMAEN0_{pr,er}}{PE0_{pr,er}} \cdot theta\_de_{pr,er,t} \cdot \left(\frac{PMAEN_{pr,er,t}}{PE_{pr,er,t}} \cdot \frac{PE0_{pr,er}}{PMAEN0_{pr,er}}\right)^{snrs2_{pr,er,t}} \\ \cdot \left[e^{\left(\sum_{ele} tge_{ele,pr,er,t}\right)\left(snrs2_{pr,er,t}-1\right)}\right] \ if\ pr = prrs \end{cases} \quad [45]$$

$$DIST_{ele,er,t} = \frac{tpxd_{ele,er,t} \cdot XD_{ele,er,t}}{tfp_{ele,er,t} \cdot tfpexo_{ele,er,t}} \quad [46]$$

$$TECH_{ele,er,t} = \frac{tpxd_{ele,er,t} \cdot XD_{ele,er,t}}{tfp_{ele,er,t} \cdot tfpexo_{ele,er,t}} \quad [47]$$

$$KLEM_{proil,er,t} = theta\_dklem_{proil,er} \cdot \frac{PDBSR0_{pr,er}}{PKLEM0_{pr,er}} \cdot XD_{pr,er,t} \cdot \left(\frac{PDBSR_{proil,er,t}}{PKLEM_{proil,er,t}} \cdot \frac{PKLEM0_{pr,er}}{PDBSR0_{pr,er}}\right)^{sn0_{proil,er,t}} \cdot \left[tfp_{proil,er,t} \cdot tfpexo_{proil,er,t}\right]^{sn0_{proil,er,t}-1} \quad [48]$$

$$KLEMRS_{prrs,er,t} = klemrs0_{prl,er} \cdot \frac{XD_{pr,er,t}}{XD0_{pr,er}} \cdot \left(\frac{PDBSR_{pr,er,t}}{PKLEMrs_{pr,er,t}} \cdot \frac{PKLEMrs0_{pr,er}}{PDBSR0_{pr,er}}\right)^{sn0_{pr,er,t}} \cdot \left[tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}\right]^{sn0_{pr,er,t}-1} \quad [49]$$

$$RESFV_{prrs,er,t} = resfv0_{prrs,er} \cdot \frac{XD_{prrs,er,t}}{XD0_{prrs,er}} \cdot \left(\frac{PDBSR_{prrs,er,t}}{PRESFV_{prrs,er,t}} \cdot \frac{PRESFV0_{prrs,er}}{PDBSR0_{prrs,er}}\right)^{sn0_{prrs,er,t}} \cdot \left[tfp_{prrs,er,t} \cdot tfpexo_{prrs,er,t}\right]^{sn0_{prrs,er,t}-1} \quad [50]$$

$$KLRS_{prrs,er,t} = \frac{KLEMRS_{prrs,er,t}}{KLEMRS0_{prrs,er}} \cdot klrs0_{prrs,er,t} \cdot \left(\frac{PKLEMRS_{prrs,er,t}}{PKLRS_{prrs,er,t}} \cdot \frac{PKLRS0_{prrs,er,t}}{PKLEMRS0_{prrs,er,t}}\right)^{snrs1_{prrs,er,t}} \quad [51]$$

$$MAEN_{prrs,er,t} = \frac{KLEMRS_{prrs,er,t}}{KLEMRS0_{prrs,er}} \cdot MAEN0_{prrs,er,t} \cdot \left(\frac{PKLEMRS_{prrs,er,t}}{PMAEN_{prrs,er,t}} \cdot \frac{PMAEN0_{prrs,er}}{PKLEMRS0_{prrs,er}}\right)^{snrs1_{prrs,er,t}} \quad [52]$$

where:

$EN_{pr,er,t}$: the demand for energy,

$PE_{pr,er,t}$: the unit cost of energy,

$ENL_{pr,er,t}$: the electricity demand by sector,

$IOV_{pr,br,er,t}$: the deliveries between branches,

$PIO_{pr,er,t}$: the input-output delivery price,

$EN_{br,er,t}$: the fuel demand by sector,

$PE_{br,er,t}$: the aggregate fuel price,

$PEUPR_{br,er,t}$: the energy price including abatement cost,

$tge_{pr,er,t}$: the technical progress on energy,

$LAV\_skld_{pr,er,t}$,$LAV\_unskld_{pr,er,t}$: the demand for skilled and unskilled labour respectively,

$PL\_skld_{pr,er,t}$, $PL\_unskld_{pr,er,t}$: the unit cost of skilled and unskilled labour,

$tgl\_skld_{pr,er,t}$, $tgl\_unskld_{pr,er,t}$: the technical progress of skilled and unskilled labour,

$MA_{pr,er,t}$: the demand for Materials,

$PM_{pr,er,t}$: the unit cost of materials,

$tgm_{pr,er,t}$: the productivity in materials,

$tge_{pr,er,t}$: the productivity in energy use.

Equations [35], [36], [37], [38] and [39] represent the demand for intermediate consumption of commodity br used in the production of sector *PR*, with $PIO_{br,er,t}$ being the unit cost of the intermediate good.

Under the above specification, the zero profit condition is always satisfied (and hence not included in the model text):

$$PD_{pr,er,t} \cdot XD_{pr,er,t} = PKLE_{pr,er,t} \cdot KLE_{pr,er,t} + PM_{pr,er,t} \cdot MA_{pr,er,t}$$

Substituting the demand functions into the production functions the unit cost functions are derived:

$$PDBSR_{pr,er,t} = PDBSR0_{pr,er} \cdot \left[theta\_dkle_{pr,er,t} \cdot \left(\frac{\frac{PKLE_{pr,er,t}}{PKLE0_{pr,er}}}{tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}} \right)^{(1-sn1_{pr,er,t})} + dm_{pr,er,t} \cdot \left(\frac{\frac{PM_{pr,er,t}}{PM0_{pr,er}}}{tfp_{pr,er,t} \cdot tfpexo_{pr,er,t}} \right)^{(1-sn1_{pr,er,t})} \right]^{\frac{1}{1-sn1_{pr,er,t}}} \quad [53]$$

$$PDBSR_{proil,er,t} = PDBSR0_{proil,er} \cdot \left[theta\_dklem_{proil,er,t} \cdot \left(\frac{\frac{PKLEM_{proil,er,t}}{PKLEM0_{proil,er}}}{tfp_{proil,er,t} \cdot tfpexo_{proil,er,t}} \right)^{(1-sn0_{proil,er,t})} + \sum_{prcrudeoil} theta\_dio_{prcrudeoil,er,t} \sum_{prcrudeoil} \left(\frac{\frac{PIO_{prcrudeoil,er,t}}{PIO0_{prcrudeoil,er}}}{tfp_{proil,er,t} \cdot tfpexo_{proil,er,t}} \right)^{(1-sn0_{proil,er,t})} \right]^{\frac{1}{1-sn0_{proil,er,t}}} \quad [54]$$

$$PDBSR_{tec,er,t} = PDBSR0_{tec,er} \cdot \left[\left(theta\_dkav_{tec,er,t} \cdot \left(\frac{\frac{PK_{tec,er,t}}{PK0_{tec,er}} \cdot e^{-tgk_{tec,er,t}}}{tfp_{tec,er,t} \cdot tfpexo_{tec,er,t}} \right) + theta\_dlskld_{tec,er,t} \cdot \left(\frac{\frac{PL_{tec,er,t}^{skld}}{PL0_{tec,er}^{skld}} \cdot e^{-tgl\_skld_{tec,er,t}}}{tfp_{tec,er,t}^{skld} \cdot tfpexo_{tec,er,t}} \right) + theta\_dlunskld_{tec,er,t} \cdot \left(\frac{\frac{PL_{tec,er,t}^{unskld}}{PL0_{tec,er}^{unskld}} \cdot e^{-tgl\_unskld_{tec,er,t}}}{tfp_{tec,er,t}^{unskld} \cdot tfpexo_{tec,er,t}} \right) \right] + \sum_{PR=ELE} \frac{theta\_dio_{pr,tec,er,t} \cdot \frac{PEUPR_{pr,tec,er,t}}{PIO0_{pr,er}}}{tfp_{tec,er,t} \cdot tfpexo_{tec,er,t}} \cdot e^{-tge(pr,tec,er,t)} + \sum_{PRE} \frac{theta\_dio_{pret,tec,er,t} \cdot \frac{PEUPR_{pr,ele,er,t}}{PIO0_{pr,er}}}{tfp_{tec,er,t} \cdot tfpexo_{tec,er,t}} \cdot e^{-tge(pr,tec,er,t)} \quad [55]$$

$$PDBSR_{ele,er,t} = dpxd_{ele,er,t} \cdot \frac{(PTECH_{ele,er,t} + PDIST_{ele,er,t})}{tfp_{ele,er,t} \cdot tfpexo_{ele,er,t}} \quad [56]$$

$$PDBSR_{prrs,er,t} = PDBSR0_{prrs,er} \cdot \left[theta\_dklem_{prrs,er,t} \cdot \left(\frac{\frac{PKLEMRS_{prrs,er,t}}{PKLEMRS0_{pr,er}}}{tfp_{prrs,er,t} \cdot tfpexo_{prrs,er,t}} \right)^{(1-sn0(prrs,er,t))} + theta\_dresf_{prrs,er,t} \cdot \left(\frac{\frac{PRESF_{prrs,er,t}}{PRESF0_{pr,er}}}{tfp_{prrs,er,t} \cdot tfpexo_{prrs,er,t}} \right)^{(1-sn0(prrs,er,t))} \right] \quad [57]$$

$$PM_{br,er,t} = PM0_{br,er} \cdot \left[\sum_{prmane} theta\_dmpr_{prmane,br,er,t} \cdot \left(\frac{PIO_{prmane,er,t}}{PIO0_{prmane,er}} \cdot e^{-tgm_{prmane,br,er,t}} \right)^{(1-sn3_{br,er,t})} \right]^{\frac{1}{(1-sn3_{br,er,t})}} \quad if\ br = prdf, prrs \quad [58]$$

$$PM_{br,er,t} = PM0_{br,er} \cdot \left[\sum_{prne=prmars} theta\_dmpr_{prmars,br,er,t} \cdot \left(\frac{PIO_{prmars,er,t}}{PIO0_{prmars,er}} \cdot e^{-tgm_{prmars,br,er,t}} \right)^{(1-sn3_{br,er,t})} \right]^{\frac{1}{(1-sn3_{br,er,t})}} \quad if\ br = proil \quad [59]$$

$$PKLE_{pr,er,t} = PKLE0_{br,er} \cdot [theta\_dkl_{pr,er,t} \cdot \frac{PKL_{pr,er,t}}{PKL0_{pr,er}}^{(1-sn2_{pr,er,t})} + deng_{pr,er,t} \cdot \frac{PENG_{pr,er,t}}{PENG0_{pr,er}}^{(1-sn2_{pr,er,t})}]^{\frac{1}{1-sn2_{pr,er,t}}} \quad [60]$$

$$PKL_{br.er.t} = PKL0_{br,er} \cdot \left[theta\_dklskld_{br.er.t} \cdot \left(\frac{PKLskld_{br.er.t}}{PKLskld0_{br.er}}\right)^{(1-sn4_{br.er.t})} + theta\_dlunskld_{br.er.t} \cdot \left(\frac{PL\_unskld_{BR,ER,t}}{PL\_unskld0_{BR,ER}} \cdot e^{-tgl_{\substack{br.er.t\\unskld}}}\right)^{(1-sn4_{br.er.t})}\right]^{1/(1-sn4_{br.er.t})} \quad [61]$$

$$PKL\_skld_{br.er.t} = PKL\_skld0_{br,er} \cdot \left[theta\_dkav_{br.er.t} \cdot \left(\frac{PK_{br.er.t}}{PK0_{br.er}} \cdot e^{-tgk_{br.er.t}}\right)^{(1-sn4_{br.er.t})} + theta\_dlskld_{br.er.t} \cdot \left(\frac{PL\_skld_{BR,ER,t}}{PL\_skld0_{br,er}} \cdot e^{-tgl_{\substack{br.er.t\\skld}}}\right)^{(1-sn4_{br.er.t})}\right]^{1/(1-sn4_{br.er.t})} \quad [62]$$

$$PENG_{pr,er,t} = PENG0_{pr,er} \cdot \left[theta\_del_{pr,er,t} \cdot \left(\frac{PEL_{pr,er,t}}{PEL0_{pr,er}} \cdot e^{-\sum_{ELE} tge_{pr,er,t}}\right)^{(1-sn5_{pr,er,t})} + theta\_de_{pr,er,t} \cdot (\frac{PE_{pr,er,t}}{PE0_{pr,er}})^{(1-sn5_{pr,er,t})}\right]^{\frac{1}{(1-sn5_{pr,er,t})}} \quad [63]$$

$$PE_{br,er,t} = \begin{cases} PE0_{pr,er} \cdot \left[\sum_{pre} theta\_depr_{pre,br,er,t} \cdot \left(\frac{PEUPR_{pre,br,er,t}}{PIO0_{pre,er}} \cdot e^{-tge_{pre,br,er,t}}\right)^{(1-sn6_{br,er,t})}\right]^{\frac{1}{(1-sn6_{br,er,t})}} & if\ de_{br,er,t} > 0 \\ 1 & if\ de_{br,er,t} = 0 \end{cases} \quad [64]$$

$$PEL_{br,er,t} = \sum_{ele} PEUPR_{ele,br,er,t} \quad [65]$$

$$PKLEM_{proil,er,t} = PKLEM0_{proil,er} \left[theta\_dkle_{proil,er,t} \cdot \left(\frac{PKLE_{proil,er,t}}{PKLE0_{proil,er}}\right)^{(1-sn1_{proil,er,t})} + theta\_dm_{proil,er,t} \cdot \left(\frac{PM_{proil,er,t}}{PM0_{proil,er}}\right)^{(1-sn1_{proil,er,t})}\right]^{\frac{1}{1-sn1_{proil,er,t}}} \quad [66]$$

$$PTECH_{ele,er,t} = \sum_{tec} theta\_dio_{tec,er,t} \cdot PIO_{tec,er,t} \quad [67]$$

$$PDIST_{ele,er,t} = PDIST0_{ele,er} \cdot \left[theta\_dkav_{ele,er,t} \cdot \left(\frac{PK_{ele,er,t}}{PK0_{ele,er}} \cdot e^{-tgk_{ele,er,t}}\right) + theta\_dlskld_{ele,er,t} \cdot \left(\frac{PL\_skld_{ele,er,t}}{PL\_skld0_{ele,er}} \cdot e^{-tgl\_skld_{ele,er,t}}\right) + theta\_dlunskld_{ele,er,t} \cdot \left(\frac{PL\_unskld_{ele,er,t}}{PL\_unskld0_{ele,er}} \cdot e^{-tgl\_unskld_{ele,er,t}}\right)\right] + \sum_{prr \neq tec,pret} theta\_dio_{prr,ele,er,t} \cdot \frac{PIO_{prr,er,t}}{PIO0_{prr,er}} + \sum_{prr \neq tec,prr=tec} theta\_dio_{pret,ele,er,t} \cdot \frac{PEUPR_{pret,ele,er,t}}{PIO0_{pret,er}} \quad [68]$$

$$PKLEMRS_{prrs,er,t} = PKLEMRS_{prrs,er,t} \cdot \left[theta\_dkl_{prrs,er,t} \cdot \frac{PKLRS_{prrs,er,t}}{PKLRS0_{prrs,er}}^{(1-snrs1_{prrs,er,t})} + theta\_dmaen_{prrs,er,t} \cdot \frac{PMAEN_{prrs,er,t}}{PMAEN0_{prrs,er,t}}^{(1-snrs1_{prrs,er,t})} \right]^{\frac{1}{1-snrs1_{prrs,er,t}}} \quad [69]$$

$$PKLRS_{prrs,er,t} = PKLRS0_{prrs,er} \cdot \left[theta\_dklskld_{prrs,er,t} \cdot \left(\frac{PKLrsskld_{prrs,er,t}}{PKLrsskld0_{prrs,er}} \right)^{(1-sn4_{prrs,er,t})} + theta\_dlunskld_{prrs,er,t} \cdot \left(\frac{PL\_unskld_{prrs,er,t}}{PL0\_unskld_{prrs,er,t}} \cdot e^{-tgl\_unskld_{prrs,er,t}} \right)^{(1-sn4_{prrs,er,t})} \right]^{\frac{1}{(1-sn4_{PRRS,ER,AN})}} \quad [70]$$

$$PMAEN_{prrs,er,t} = PMAEN0_{prrs,er} \cdot \left[theta\_dm_{prrs,er,t} \cdot \left(\frac{PM_{prrs,er,t}}{PM0_{prrs,er}} \right)^{(1-snrs2_{prrs,er,t})} + theta\_de_{prrs,er,t} \cdot \left(\frac{PE_{prrs,er,t}}{PE0_{prrs,er}} \right)^{(1-snrs2_{prrs,er,t})} + theta\_del_{prrs,er,t} \cdot \left(\frac{PEL_{prrs,er,t}}{PEL0_{prrs,er}} e^{-\sum_{ele} tge_{ele,prrs,t}} \right)^{(1-snrs2_{prrs,er,t})} \right]^{\frac{1}{(1-snrs2(prrs,er,t))}} \quad [71]$$

$$PKLrs_{\substack{prrs,er,t}} = PKLrs0_{\substack{prrs,er \\ skld}} \cdot \left[theta\_dkav_{prrs,er,t} \cdot \left(\frac{PK_{prrs,er,t}}{PK0_{prrs,er}} \right)^{(1-sn4_{prrs,er,t})} + theta\_dlskld_{prrs,er,t} \cdot \left(\frac{PL_{\substack{prrs,er,t \\ skld}}}{PL0_{\substack{prrs,er,t \\ skld}}} \right)^{(1-sn4_{prrs,er,t})} \right]^{\frac{1}{(1-sn4(prrs,er,t))}} \quad [72]$$

For the depletable resource sectors reserves are considered to be a discrete production factor. The international price of the fossil fuel is calculated so as to balance total supply and total demand. Reserves are subject to depletion at an exogenous growth rate (growth RES_t). The exogenous growth rate is calculated based on the remaining reserves, the production of fossil fuels and the yet to find reserves.

$$\left(\sum_{er} RESFV0_{prrs,er} \right) \cdot (1 + growth\_RES_t)^{period} = \sum_{er} RESFV_{prrs,er,t} \quad [73]$$

$$PRESF_{prrs,er,t} = PWRESF_{prrs,t} \quad [74]$$

Where:

$PWRESF_{prrs,er}$: the international price of fossil fuel,

$RESFV0_{prrs,er,t}$: exogenous reserves of fossil fuels at the base year.

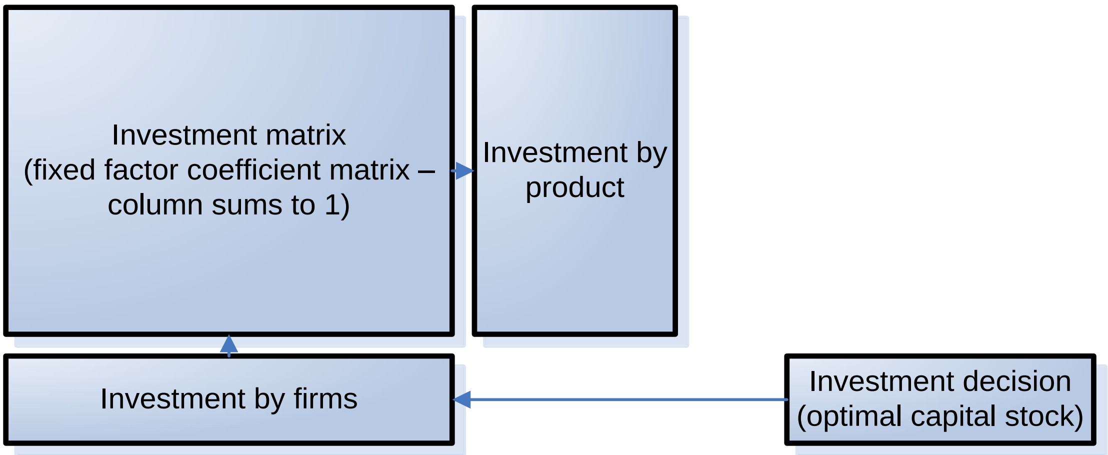
period: is the time interval between two simulation years

4.3 Investment demand

GEM-E3 is a recursive dynamic model (solved sequentially over time). The sequential equilibria are linked through a motion equation regarding the update of the capital stock. According to the standard neoclassical approach agents investment decision depends on the rental cost of capital in the presence of adjustment costs and on its replacement cost. In GEM-E3 agents have myopic expectations. Their future planning is based on current prices. It is assumed that investment that takes place in time t increases the production capacity at time t+1.

Figure 9 illustrates the investment decisions of the firm in the GEM-E3 model. The basic methodological approaches to investment specification include the accelerator model (AM)[14] and *q* of Tobin (1969)[15].

Figure 9: Investment decision of firms



The law of motion of capital stock is:

$$KAVC_{pr,er,t} = \left(1 - d_{pr,er,t}\right) \cdot KAVC_{pr,er,t-1} + INVV_{pr,er,t}$$

where:

$KAVC_{pr,er,t}$: capital stock by branch,

$d_{pr,er,t}$: depreciation rate,

---

[14] AM assume that optimal demand of capital is a function of the production level $KAVC^{*}_{pr} = \mu \cdot XD_{pr}$. Prices, wages and interest rate have no effects on the formation of capital demand. Thus since the model assumes immediate adjustment of capital to the optimal level, investment is also a direct function of the production level : $INVV_{pr,t} = KAVC^{*}_{pr,t} - KAVC^{*}_{pr,t-1} = \mu \cdot (XD_{pr,t} - XD_{pr,t-1})$. An alternative to this approach regards the non-automatic capital adjustment $INVV_{PR,T} = \lambda \cdot \left(KAVC^{*}_{pr,t} - KAVC^{*}_{pr,t-1}\right)$

[15] According to this approach, net investment depends on the relationship between the market price of the capital good and its replacement cost.

$KAVC_{pr,er,t-1}$: capital stock of the previous period,

$INVV_{pr,er,t}$: investment by firm in volume.

Investment covers the change in firm's potential plus the capital depreciation. Using the average Tobin's *q* according to Hayashi (1982) the firm decides the optimal level of investment according to the rental price of capital and its replacement cost ($\frac{PK_{pr,er,t}}{PINV_{pr,er,t} \cdot (rr_{er,t} + d_{pr,er,t})}$). It is also assumed that the firms always replace the depreciated capital ($d_{pr,er,t} \cdot KAVC_{pr,er,t}$). Hence the investment function becomes:

$$INVV_{pr,er,t} = KAV^*_{pr,er,t} \cdot \left[\frac{PK_{pr,er,t}}{PINV_{pr,er,t} \cdot (rr_{er,t} + d_{pr,er,t})} - 1 + d_{pr,er,t}\right]$$

This function is modified in order to take into account: i) adjustment/instalment investment costs (*a0inv)*, ii) flexibility to replace capital (sn4), iii) speed of adjustment (*a1inv)*, iv) exogenous firm's expectations on future profitability (*stgr)* and v) productivity of capital. The investment function entering the model is [75].

$$INVV_{pr,er,t} = \begin{cases} a0inv_{pr,er,t} \cdot KAV_{pr,er,t} \cdot \left[\left(\frac{PK_{pr,er,t}}{PINV_{pr,er,t} \cdot (RLTLREU_{er,t} \cdot RLTLR_{er,t} \cdot RLTLRWORLD_{er,t} + decl_{pr,er,t-1})}\right)^{sn4_{pr,er,t} \cdot a1inv_{pr}} (1 + stgr_{pr,er,t}) - 1 + decl_{pr,er,t-1}\right] & if\ er = euc27 \\ a0inv_{pr,er,t} \cdot KAV_{pr,er,t} \cdot \left[\left(\frac{PK_{pr,er,t}}{PINV_{pr,er,t} \cdot (RLTLR_{er,t} \cdot RLTLRWORLD_{er,t} + decl_{pr,er,t-1})}\right)^{sn4_{pr,er,t} \cdot a1inv_{pr}} (1 + stgr_{pr,er,t}) - 1 + decl_{pr,er,t-1}\right] & if\ er \neq euc27 \end{cases} \quad [75]$$

where:

$PK_{pr,er,t}$: the user cost of capital,

$PINV_{pr,er,t}$: the price of investment,

$decl_{pr,er,t-1}$: the depreciation rate of the previous period,

$stgr_{pr,er,t}$: the expected growth rate of the sector.

$a0inv_{pr,er,t}$ and $a1inv_{pr,er,t}$ regard capital adjustment and price elasticity respectively ($a1inv_{pr,er,t}$ is the respective λ value of the accelerator model when capital does not adjust immediately). Investment increases the production potentials of the firm from the following period. The unit cost of capital results as the dual price of the equilibrium function of the available and the demanded capital stock.

Firms' investment is translated into demand for investment goods which are produced from the rest of the sectors of the economy through an investment matrix of constant coefficients
$tinvpv_{pr,br}$:

$$INVPV_{pr,br,er,t} = tinvpv_{pr,br,er,t} \cdot \frac{INVV_{br,er,t}}{INVV0_{br,er}} \cdot INVPV0_{pr,br,er} \quad [76]$$

The next period capital stock is given by the equation:

$$KAVC_{pr,er,t} = \left\{ \left(1 - decl_{pr,er,t}\right)^{PERIOD} \cdot KAV_{pr,er,t} + \left(\frac{1 - \left(1 - decl_{pr,er,t}\right)^{PERIOD}}{decl_{pr,er,t}} \right) \cdot INVV_{pr,er,t} \right. \qquad [77]$$

Since the capital is fixed within each period, the investment decision of the firms affects their production frontier only in the next period.

The investment demand of each branch is transformed into a demand by product, through fixed technical coefficients, derived from an investment matrix by product and ownership branch. The investment matrix is computed using the intermediate goods used in the production of capital goods and data provided in the literature on the inputs delivered by the sectors of the economy to the investments undertaken by each sector of production. The standard approach when no additional data are available is to use the same coefficient structure for each branch. This approach can be extended when additional information is available on investment by branch and on the structure of capital formation. In order to make changes in the investment matrix a simple procedure is followed. The initial investment matrix (with the same coefficients in each branch) is updated with the new investment shares Then a RAS procedure is followed in order to ensure that the total of each row and column of the investment matrix remains constant and that the model remains balanced.

4.4 Government behaviour

The Government's behaviour is exogenous in GEM-E3. Government's final demand (*$GCV_{pr,er,t}$*) by product is obtained by applying fixed coefficients (*$tgcv_{pr,er,t}$*) to the exogenous volume of government consumption (*$GCTV_{er,t}$*):

$$GCV_{pr,er,t} = GCTV_{er,t} \cdot tgcv_{pr,er,t} \quad if\ swGC = 0 \qquad [78]$$

$$GCV_{pr,er,t} = sh\_gctv_{er,t} \cdot [\sum_{br} PHC0_{br,er} \cdot HCV_{br,er,t} + \sum_{br}\left(PGC0_{br,er} \cdot GCV_{br,er,t} + \sum_{prr} PINVP0_{prr,br,er} \cdot INVPV_{prr,br,er,t} + \sum_{cr} PWE0_{br,er} \cdot EXPO_{br,er,cr,t} - PIMP0_{br,er} \cdot IMP_{br,er,t}\right)] \cdot tgc \qquad [79]$$

Where:

$sh\_gctv_{er,t}$: coefficient that relates government consumption with GDP evolution in case where swGC switch is activated

$INVPV_{prr,br,er,t}$: investment matrix,

$PINVP0_{prr,br,er}$: price of deliveries to investment in the base year,

$EXPO_{br,er,cr,tt}$: bilateral exports,

$PWE0_{br,er}$: price of exports in the base year,

$IMP_{br,er,t}$: imports,

$PIMP0_{br,er}$: price of imports in the base year,

$PGC0_{br,er}$: price of government consumption in the base year,

$PHC0_{br,er}$: price of household consumption in the base year,

br and *prr* are sets aliased with *pr*. *swGC* is the switch parameter which allows for endogenous computation of government consumption.

Public investment, assumed exogenous in the model, is performed by the branch of non-market services. Transfers to the households are computed as an exogenous rate per head times the population.

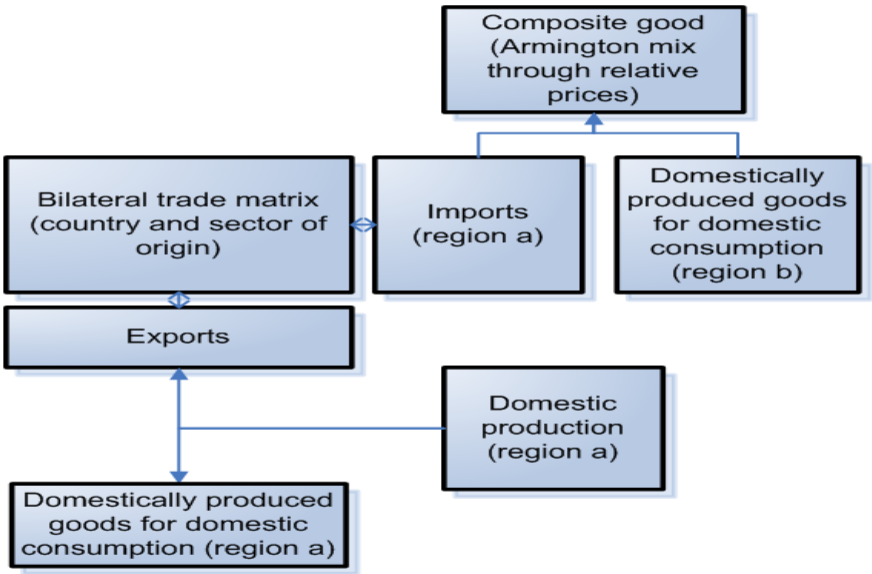
On the receipt side, the model distinguishes between 9 categories of receipts namely: i) indirect taxes, ii) environmental taxes, iii) direct taxes, iv) value added taxes, v) production subsidies, vi) social security contributions, vii) import duties, viii) foreign transfers and ix) government firms. These receipts are coming from product sales (i.e. from branches) and from sectors (i.e. agents). The receipts from product sales in value (*FG*), which include indirect taxes, the VAT, subsidies and duties, are computed from the corresponding receipts in value, given the tax base and the tax rate. The receipts from agents are computed from the tax base and the tax rate (social security contributions, direct taxation), share of government in total capital income (for government firm's income) or exogenous (transfers from and to the ROW).

4.5 Domestic demand and trade flows

The demand of products by the consumers, the producers (for intermediate consumption and investment) and the public sector constitutes the total domestic demand. This total demand[16] is allocated between domestic products and imported products, following the Armington specification. In this specification, branches and sectors use a composite commodity which combines domestically produced and imported goods, which are considered as imperfect substitutes (Armington assumption).

Each country buys and imports at the prices set by the supplying countries following their export supply behaviour. The buyer of the composite good (domestic) seeks to minimise his total cost and decides the mix of imported and domestic products so that the marginal rate of substitution equals the ratio of domestic to imported product prices.

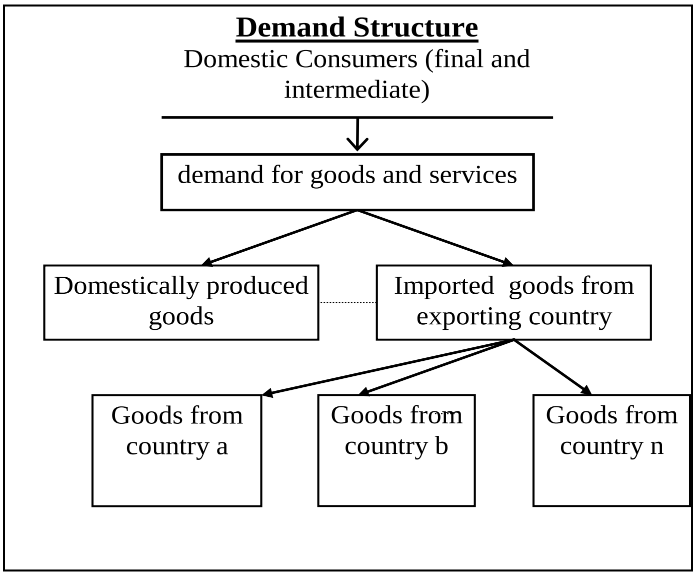
Figure 10: Trade matrix for EU and the rest of the world



GEM-E3 employs a nested commodity aggregation hierarchy, in which branch's i total demand is modelled as demand for a composite good or quantity index Y_i (Figure 10) which is defined over demand for the domestically produced variant (XXD_i) and the aggregate import good (IMP_i). At a next level, demand for imports is allocated across imported goods by country of origin (Figure 11). Bilateral trade flows are thus treated endogenously in GEM-E3.

[16] In the GEM-E3 model it is assumed that the buyer's decision is uniform throughout the economy, therefore the Armington specification is applied at the level of total domestic demand for each sector.

Figure 11: Domestic demand and trade flows nesting scheme



The minimum unit cost of the composite good determines its selling price. This is formulated through a CET unit cost function, involving the selling price of the domestic good, which is determined by goods market equilibrium, and the price of imported goods, which is taken from the 2nd level Armington. By applying Shephard's lemma, total demand for domestically produced goods and for imported goods is derived.

In particular the cost minimization problem (for the 1st level) is:

$$\min C_{pr,er,t} = PXD_{pr,er,t} \cdot XXD_{pr,er,t} + PIMP_{pr,er,t} \cdot IMP_{pr,er,t}$$

where:

$PXD_{pr,er,t}$: price of domestically produced good,

$XXD_{pr,er,t}$: production for domestic use,

$PIMP_{pr,er,t}$: import price,

$IMP_{pr,er,t}$: imports.

such that:

$$Y_{pr,er,t} = AC_{pr,er,t} \cdot \left[\delta_{pr,er,t} \cdot XXD_{pr,er,t}^{\frac{\sigma_{x_{pr,er,t}}-1}{\sigma_{x_{pr,er,t}}}} + \left(1 - \delta_{pr,er,t}\right) \cdot IMP_{pr,er,t}^{\frac{\sigma_{x_{pr,er,t}}-1}{\sigma_{x_{pr,er,t}}}} \right]^{\frac{\sigma_{x_{pr,er,t}}}{\sigma_{x_{pr,er,t}}-1}}$$

where:

$Y_{pr,er,t}$: composite good,

$AC_{pr,er,t}$: scale parameter in the Armington function,

$\delta_{pr,er,t}$: share parameter estimated from the base year data related with the value shares of $XXD_{pr,er,t}$ and $IMP_{pr,er,t}$ in the demand for composite good $Y_{pr,er,t}$,

σ_x: the Armington elasticity between imported and domestically produced goods.

The optimal demand for domestic and imported goods is obtained by employing the Shephard's lemma.

$$\mathrm{XXD}_{\mathrm{pr,er,t}} = \begin{cases} \mathrm{Y}_{\mathrm{pr,er,t}} \cdot \mathrm{AC}_{\mathrm{pr,er,t}}^{\sigma_{x_{\mathrm{pr,er,t}}}-1} \cdot \left(1-\delta_{\mathrm{pr,er,t}}\right)^{\sigma_{x_{\mathrm{pr,er,t}}}} \cdot \left(\dfrac{\mathrm{PY}_{\mathrm{pr,er,t}}}{\mathrm{PXD}_{\mathrm{pr,er,t}}}\right)^{\sigma_{x_{\mathrm{pr,er,t}}}} & \text{if } \mathrm{AC}_{\mathrm{pr,er,t}} \neq 0 \\ \mathrm{Y}_{\mathrm{pr,er,t}} & \text{if } \mathrm{AC}_{\mathrm{pr,er,t}} = 0 \end{cases} \quad [80]$$

$$IMPC_{pr,er,t} = Y_{pr,er,t} \cdot AC_{pr,er,t}^{\sigma_{x_{pr,er,t}}-1} \cdot \delta_{pr,er,t}^{\sigma_{x,pr,er,t}} \cdot \left(\frac{PY_{pr,er,t}}{PIMP_{pr,er,t}}\right)^{\sigma_{x_{pr,er,t}}} \quad [81]$$

where:

$IMPC_{pr,er,t}$: the competitive imports by branch,

$PY_{pr,er,t}$: the unit cost for the composite good.

$$PY_{pr,er,t} = \begin{cases} \dfrac{1}{AC_{pr,er,t}} \cdot \left[\delta_{pr,er,t}^{\sigma_{x,pr,er,t}} \cdot PIMP_{pr,er,t}^{1-\sigma_{x_{pr,er,t}}} \cdot + \left(1-\delta_{pr,er,t}\right)^{\sigma_{x_{pr,eu,t}}} \cdot PXD_{pr,er,t}^{1-\sigma_{x_{pr,er,t}}}\right]^{\frac{1}{1-\sigma_{x_{pr,er,t}}}} & if\ pr \neq brt, theta\_dkav \neq 0 \\ PIMP_{pr,er,t} \quad if\ theta\_dkav = 0 & \end{cases} \quad [82]$$

$$PY_{pr,er,t} = PIMP_{pr,er,t} \cdot rtxd_{pr,er,t} + PD_{pr,er,t} \cdot txsub_{pr,er,t} \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \left(1-rtxd_{pr,er,t}\right) \quad if\ pr = brnt, theta\_dkav \neq 0 \quad [83]$$

where:

$PIMP_{pr}$: the price of imported good *PR* computed as an average of the overall trading partners,

$rtxd_{pr,er,t}$: the parameter indicating the share of imports in total domestic demand of non - traded goods,

$txsub_{pr,er,t}$: the subsidy rate

brt: traded branches,

brnt: non-traded branches,

$theta\_dkav_{pr,er,t}$: value share parameter of capital l in the aggregate (KLskld) bundle or in the production.

Equations [80], [81] derive from the Armington equation, i.e. the assumption on imperfect substitution of domestic and imported goods, and thereby refer only to tradable goods. The term "tradable" is now used to express that the Armington assumption stands for these specific goods and does not mean that the "non-traded" goods (*brnt*) are not imported or exported but instead that they are not considered as substitutes to domestic goods.

Where total imports by branch in volume terms are given as follows:

$$IMP_{pr,er,t} = IMPC_{pr,er,t} + IMPNC_{pr,er,t} \quad \text{for pr=brt} \qquad [84]$$

Equation [84] indicates that imports of tradable goods are the sum of competitive imports, deriving from the Armington equation and the non-competitive imports. Non-competitive imports by branch are given as a fixed share of domestic production:

$$IMPNC_{pr,er,t} = rtnc_{pr,er,t} \cdot XD_{pr,er,t} \quad \text{for pr} = \text{brt} \qquad [85]$$

$$IMP_{pr,er,t} = rtxd_{pr,er,t} \cdot Y_{pr,er,t} \quad \text{for pr} = \text{brnt and theta\_dkav} \neq 0 \qquad [86]$$

$$IMP_{pr,er,t} = Y_{pr,er,t} + \sum_{er} EXPO_{pr,cr,er,t} \quad \text{for pr} = \text{brnt and theta\_dkav} = 0 \qquad [87]$$

where:

$rtnc_{pr,er,t}$: the share (fixed) of non-competitive imports per unit of production

The equation above indicates that imports of non-tradable goods are a fixed share of total domestic demand, while imports of non-tradable goods that are not domestically produced (i.e. *thetaDKAV=0*) must be equal to total domestic supply and to total exports of the good.

At the 2nd level, import demand is allocated across countries of origin using again a CET functional form.

$$PIMP_{pr,er,t} = \left[\sum_{cr} beta_{pr,er,cr,t}^{sigmai_{pr,er,t}} \cdot PWXO_{pr,er,cr,t}{}^{(1-sigmai_{pr,er,t})}\right]^{\left(\frac{1}{1-sigmai_{pr,er,t}}\right)} \qquad [88]$$

where:

$PIMP_{pr,er,t}$: the price of total imports of good PR demanded by country ER,

$beta_{pr,er,cr,t}$: the share parameter for Armington,

$sigma_{pr,er,t}$: the elasticity of substitution,

$PWXO_{pr,er,cr,t}$: denotes import price of good *PR* for country *EU* originating from country *CR* :

$$PWXO_{pr,cs,cr,t} = PWE_{pr,cr,t} + txduto_{pr,cs,cr,t} \cdot \frac{PCI_{cs,t}}{PCIBASE_{cs,t}} + \sum_{itrn} \left(cif\_vtwr_{itrn,pr,cs,t} \cdot PTR_{itrn,t}\right) \qquad [89]$$

where:

$PWE_{pr,cr,t}$: the export price in international currency,

$cif\_vtwr_{,itrn,pr,cs,t}$: the demand share for transport margins,

$PTR_{itrn,t,}$: the international transport margin price.

The GEM-E3 model distinguishes between three types of transport services, namely water, air and inland. The international transport margin price is determined by the following equation:

$$PTR_{itrn,t} > \sum_{er}\left(thetavst_{itrn,er,t} \cdot \frac{PWE_{pr,er,t}}{PWE0_{pr,er}}\right) \qquad [90]$$

Where:

$thetavst_{itrn,er,t}$: measures the share of each country in total international transport margins in the base year. The activity level of each type of transport is defined as

$$YVST_{itrn,t} \cdot vtag_{itrn,t} > \sum_{br,er,cr}\left(EXPO_{pr,er,cr,t} \cdot cif\_vtwr_{itrn,pr,er,cs,t}\right) \qquad [91]$$

$vtag_{itrn,t}$: the output per type of transport in the international pool in the base year

Exports of transport services are given by:

$$\sum\nolimits_{cr,cr,br} EXPO_{br,cr,cs,t} \cdot cif\_vtwr_{itrn,br,cr,cs,t} < YVST_{itrn,t} \cdot vtag_{itrn,t} \qquad [92]$$

The bilateral import price equals the export price of the exporter in case of tradable services, while in case of merchandise sectors the bilateral import price is given by the export price plus the bilateral cif/fob margins.

Thereby, the equation to estimate bilateral imports derives from the second level of the CET function taking into account the bilateral import prices in order to estimate the optimum bundle of imports originating from each country.

In particular, for computing $IMPO_{br,cr,cs,t}$:

$$\frac{IMPO_{pr,er,co,t}}{IMP_{pr,er,t}} = \frac{\partial PIMP_{pr,er,t}}{\partial PIMPO_{pr,er,co,t}} \ \forall er \neq rw \ computing \ IMPO_{pr,er,co,t}$$

where:

$IMPO_{br,cr,cs,t}$: denotes imports of good *pr* demanded by country *eu* from country *co*.

$$IMPO_{br,cr,cs,t}{=}IMP_{nr,cr,t} \cdot \left(beta_{br,cr,cs,t} \cdot \frac{PIMP_{br,cr,t}}{PWXO_{br,cr,cs,t}}\right)^{sigmai_{br,cr,t}} \qquad [93]$$

Bilateral exports are then given in order to satisfy the Walras law by equating the exports of sector *pr* of country *co* to country *er* with the imports of sector *pr* of country *er* from country *co*.

Export of services from country *cr* to country *cs* will be equal to the bilateral import of services of country *cs* from *cr*. The model ensures analytically that, under the above assumptions, the balance of trade matrix in value and the global Walras law is verified in all cases. A trade flow from one country to another country matches, by construction, the inverse flow. The model ensures this symmetry in volume, value and deflator. Thus the model guarantees (in any scenario run) all balance conditions applied to the world trade matrix, as well as the Walras law at the level of the planet.

4.6 Current account instruments

The model allows for a free variation of the balance of payments, while the real interest rate is kept fixed. An alternative approach, implemented in the GEM-E3 model as an option, is to set the current account of a country or of the total EU with the rest of the world (RoW) to a pre-specified value, in fact a time-series set of values, expressed as percentage of GDP. This value is obtained either as a result from the baseline scenario or is given by the modeller as a share of GDP through the parameter $share\_ca_{er,t}$. As a shadow price of this constraint, a shift of the real interest rate at the level of the EU is endogenously computed. This shift is proportionally applied to the real interest rates of each member-state.

This mechanism enables a robust comparison between scenarios since the modeller does not allow for additional borrowing/lending (in GEME-E3 borrowing/lending is in real terms the balance of trade) of the country due to scenario policies but instead allows for an endogenous change of the real interest rate of the country/region. For example, in a climate policy scenario with a fixed current account as a share of GDP (fixed in baseline levels), the country/region under constraint cannot increase its imports as a reaction to increased unit cost of energy and thereby sustain levels of consumption and welfare but instead has to face an increased real interest rate.

The option of a constant current account as a percentage of GDP is activated in the model by a switch parameter. In order to sustain the current account as a share of GDP in baseline levels for a country, the respective switch parameter to be activated is $swonca_{er,t}$, while in order to achieve the same constraint for the aggregate EU the respective switch parameter is $swoncaeu_t$. The respective equations are [94], [95], while equation [96] is to be activated in order to obtain a pre-assumed share of current account to GDP (equal to $share\_ca_{er,t}$). The switch parameter for this equation is $swoncafix_{er,t}$.

$$SURPL_{se,er,t} = share\_ca_{er,t} \cdot VU_{er,t} \quad se = w, swoncafix_{er,t} \neq 0 \qquad [94]$$

$$SURPL_{se,er,t} = surplwrrffx_{er,t} \cdot VU_{er,t} \quad se = w, swoncaeu_{er,t} \neq 0 \qquad [95]$$

$$\sum_{er} SURPL_{se,er,t} = surplwrffxer_t \cdot \sum_{er} VU_{er,t} \quad se = w, er = euc27, swoncaeu_{er,t} \neq 0 \qquad [96]$$

where:

$SURPL_{s,er,t}$: the surplus of the country with the rest of the world (namely the balance of trade)

$surplwrrffx_{er,t}$: the share of current account in gross value added in the baseline scenario.

$surplwrrffxeu_{er,t}$: the share of current account on the *EU* region in gross value added in the baseline scenario.

$VU_{er,t}$: the gross value added of the country is given by:

$$VU_{er,t} = \sum_{br} VA_{fa,br,er,t} \qquad [97]$$

4.7 Institutional transfers

The only direct transfers and value flows between branches and sectors in the model, refer to government revenue/expenditures through taxes/subsidies and world revenue/expenditures through imports/exports. Flows considered as revenues of branches (in fact product demand) coming from sectors are detailed in: final consumption of products by sector in value, which includes exports, investment by product and sector in value and stock variation in value.

The following equation describes all tax revenues and subsidy expenditure of the government disaggregated by government revenue categories:

$$FGRB_{gvb,pr,er,t} = \sum_{cr} txduto_{pr,er,cr,t} \cdot IMPO_{pr,er,cr,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}$$

GVB=duties [98]

$$FGRB_{gvb,pr,er,t} = txsub_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot XD_{pr,er,t}$$

GVB=subsidies [99]

$$FGRB_{gvb,pr,er,t} = txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \left[\sum_{br} \left(IOV_{pr,br,er,t} + ABIOV_{pr,br,er,t}\right) + GCV_{pr,er,t} + HCV_{pr,er,t} + \sum_{br}\left(INVPV_{pr,br,er,t}\right) + \sum_{br}\left(IMAT\_FLOW_{pr,br,er,t}\right) + \left(EFFI\_FLOW\_H_{er,t} \cdot nrgeffi\_bcap\_h_{pr,t}\right)\right]$$

GVB=Indirect taxes [100]

$$FGRB_{gvb,pr,er,t} = txvat_{pr,er,t} \cdot PY_{pr,er,t} \cdot \left(1 + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}\right) \cdot \left(HCV_{pr,er,t} + GCV_{pr,er,t}\right)$$

GVB=Value added tax [101]

$$FGRB_{gvb,pr,er,t} = txvat_{pr,er,t} \cdot \left[PY_{pr,er,t} \cdot \left(1 + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}\right) \cdot \left(HCV_{pr,er,t} + GCV_{pr,er,t}\right) + \sum_{br} \left[PY_{pr,er,t} \cdot \left(1 + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}\right) \cdot INVPV_{pr,br,er,t} \right] + \sum_{br} \left[PY_{pr,er,t} \cdot \left(1 + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}\right) \cdot IMAT\_FLOW_{pr,br,er,t} \right] + PY_{pr,er,t} \cdot EFFI\_FLOW\_H_{er,t} \cdot nrgeffi\_bcap\_h_{pr,t} \cdot \left(1 + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}}\right) \right]$$

GVB=Value added tax (pr=cns) [102]

$$FGRB_{gvb,pr,er,t} = \sum_{po1} \left(TXENV_{po1,pr,er,t} \cdot EMMBR_{po1,pr,er,t} \cdot \left(1 - swonpor_{po1,pr,er,t}\right)\right) + \sum_{po1} \left(BUSAT_{po1,pr,er,t} + SHAUCTBR_{po1,pr,er,t} \cdot SALEP_{po1,pr,er,t} \right) + tx\_effix_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \sum_{pret} IOV_{pret,pr,er,t,} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pr,er,t}$$

GVB=Environmental tax (swtxexobr=0, swonpor=0) [103]

$$FGRB_{gvb,pr,er,t} = \sum_{po1} \left(TXENV_{po1,pr,er,t} \cdot EMMBR_{po1,pr,er,t} \cdot \left(1 - swonpor_{po1,pr,er,t}\right)\right) + \sum_{po1} \left(BUSAT_{po1,pr,er,t} + SHAUCTBR_{po1,pr,er,t} \cdot SALEP_{po1,pr,er,t} \right) + \sum_{po1} \left(TXEMEU_{po1,pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot EMMBR_{poi,pr,er,t} \right) + tx\_effix_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \sum_{pret} IOV_{pret,pr,er,t,} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pr,er,t}$$

GVB=Environmental tax (swtxexobr=1, swonpor=1) [104]

$$FGRB_{gvb,pr,er,t} = \sum_{po1} \left(TXENV_{po1,pr,er,t} \cdot EMMBR_{po1,pr,er,t} \cdot \left(1 - swonpor_{po1,pr,er,t}\right)\right) + \sum_{po1} \left(BUSAT_{po1,pr,er,t} + SHAUCTBR_{po1,pr,er,t} \cdot SALEP_{po1,pr,er,t} \right) + \sum_{poi} \left(TXEM_{po1,pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot EMMBR_{po1,pr,er,t} \right) + tx\_effix_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \sum_{pret} IOV_{pret,pr,er,t,} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pr,er,t}$$

GVB=Environmental tax (pr=pre, pre≠pr, swtxexobr=1, swonpor=1) [105]

$$FGRB_{gvb,pr,er,t} = \sum_{po1} \left(TXENV_{po1,pr,er,t} \cdot EMMBR_{po1,pr,er,t} \cdot \left(1 - swonpor_{po1,pr,er,t}\right)\right) + \sum_{po1} \left(BUSAT_{po1,pr,er,t} + SHAUCTBR_{po1,pr,er,t} \cdot SALEP_{po1,pr,er,t} \right) + tx\_effix_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \sum_{pret} IOV_{pret,pr,er,t,} + tx\_effi\_H_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pr,er,t} + \sum_{lnd} \left[thcfv_{pr,lnd,er,t} \cdot \frac{\sum_{dg} \sum_{po1} \left(TXENVHDG_{po1,dg,er,t} \cdot EMMHLND_{po1,lnd,dg,er,t} \right)}{\sum_{pre} thcfv_{pre,lnd,er,t}} \right]$$

GVB=Environmental tax (swtxexobr=0, swonpor=0 and $\sum_{pre} thcfv_{pre,lnd,er,t} \neq 0$) [106]

where:

$txduto_{pr,er,cr,t}$: bilateral duty rate,

$txsub_{pr,er,t}$: the subsidy rate,

$txit_{pr,er,t}$: the indirect tax rate,

$PY_{pr,er,t}$: the price of domestic demand,

$HCV_{pr,er,t}$: the deliveries to private consumption.

$IMAT\_FLOW_{pr,br,er,t:}$ investment matrix for building the energy saving equipment

$EFFI\_FLOW\_H_{er,t}$: Household expenditure on energy saving

$nrgeffi\_bcap\_h_{pr,t}$: Building materials for energy saving

$PINVP_{pr,er,t}$: the price of deliveries to investment,

$txvat_{pr,er,t}$: VAT rate per branch,

$TXENV_{pr,er,t}$: the environmental tax,

$EMMBR_{po1,pr,er,t}$: the emissions by branches,

$swonpor_{po1,pr,er}$: the switch for club participation,

$BUSAT_{po1,pr,er,t}$: the expenditures or receipts on permits,

$shauctbr_{po1,pr,er,t}$: the share of auctioned permits,

$SALEP_{po1,pr,er,t}$: the value of endowment in permits.

$tx\_effix_{po1,pr,er,t}$: the energy tax rate imposed on firms

$tx\_effi\_h_{pr,er,t}$: the energy tax rate imposed on household

$PCI_{er,t}$: the price index for private consumption,

$PCIBASE_{er,t}$: the private consumption price in the base year,

$Thcfv_{pr,lnd,er,t}$: the share of branch in the delivery of private consumption,

$TXENVHDG_{po1,dg,er,t}$: the environmental tax,

$EMMHLND_{PO1,,lnd,dg,er,t}$: the emissions of household for durable and linked non-durable.

4.7.1 Transfers between sectors

The transfers between sectors include income flows as described in the Social Accounting Matrix and are described by the following equations in GEM-E3 model. These transfers formulate the disposable income of the households. The most important of these transfers include:

- The dividends the firms pay to the households, which is proportional to the net revenues of the firms [107]
- The social benefits that the government pays to the households, which depends on the number of employees by branch and the rate of government payments to the unemployed [108]
- The direct taxes on the firms which is again proportional to the net revenues of the firms (now excluding dividends) and the households, where the tax is proportional to their disposable income [112]
- The payments of individuals to the government for social security [111]

$$FSESE_{se,sr,er,t} = txdividh_{er,t} \cdot \sum_{fa} FSEFA_{se,sr,er,t} + \sum_{sei} FSESE_{sr,sei,er,t} - FC_{sr,er,t}$$

Firms pay/ households receive (se:=h), (sr:= f) [107]

$$FSESE_{se,sr,er,t} = \left(txsocbenh_{er,t} + TRHOUS_{er,t}\right) \cdot actp\_t_{er,t} \cdot PCI_{er,t} + \sum_{po1}\sum_{lnd}\left(1 - SHAUCTH_{po1,er,t}\right) \cdot SALEPH_{po1,lnd,er,t} + 0.5 \cdot \sum_{po1} SALEP_{po1,er,t}$$

Government pays/ households receive (se:=h), (sr:=g) [108]

$$FSESE_{se,sr,er,t} = \sum_{gv} FGRS_{gv,sr,er,t}$$

Government pays/Government receives (se:=g), (sr:=g) [109]

$$FSESE_{se,sr,er,t} = \sum_{po1}\sum_{br} BUSAT_{poi,br,er,t} + \sum_{po1}\sum_{lnd} BUSATH_{po1,lnd,er,t} - \sum_{po1} 0.5 \cdot salep_{po1,er,t}$$

Government pays/World receives (se:=w), (sr:=g) [110]

$$FGRS_{gvs,se,er,t} = \sum_{br}\left(\left(txfss_{br,er,t} - idea_{er,t}\right) \cdot VA_{fa,br,er,t}\right)$$

Household pays social security (gvs:=ss), (se:=h), [111]

(fa:=l)

$$FGRS_{gvs,se,er,t} = txdirtaxf_{er,t} \cdot \left(\sum_{fa} FSEFA_{se,fa,er,t} + \sum_{sr} FSESE_{se,sr,er,t} \right)$$ Firm pays direct taxes (gvs:=dt), (se:=f) [112]

where:

$txdividh_{er,t}$: the rate of dividend from firms to household,

$FSEFA_{se,sr,er,t}$: the payments by factors to the sectors,

$FSESE_{se,sr,er,t}$: the transfers between sectors,

$FC_{sr,er.t}$: the consumption by sector,

$txsocbenh_{er,t}$: social benefits rate,

$TRHOUS_{er,t}$: the increase in social benefit transfers (scenarios),

$actp\_t_{er,t}$: the active population.

$SHAUCTH_{po1,er,t}$: the share of auctioned permits per household,

$SALEPH_{po1,lnd,er,t}$: the value of endowment of permits for households,

$SALEP_{po1,er,t}$: the value of endowment of permits for firms,

$BUSAT_{po1,br,er,t}$: the expenditure of firms for buying permits

$BUSATH_{po1,br,er,t}$: the expenditure of households for buying permits

$FGRS_{gvs,se,er,t}$: the payments by sectors to public sector expenditure categories,

$txfss_{br,er,t}$: the social security rate,

$IDEA_{er,t}$: the endogenous reduction in social security rates (scenarios)

$txdirtaxf_{er,t}$: the rate of direct taxes on firms.

The transfers between factors of production and the economic sectors as given in the Social Accounting Matrix are described in the equations below. The most important of these transfers include:

- Revenues of sectors coming from factors , e.g. labour income of households. Flows considered as revenues of factors coming from branches represent the value added, in value

- Flows from factors to factors and from factors to branches are equal to zero
- Factor payments to sectors are coming from value added and distributed according to an exogenous structure

$VA_{fa,pr,er,t} = LAV_{\substack{pr,er,t \\ skld}} \cdot PL_{\substack{pr,er,t \\ skld}} + LAV_{\substack{pr,er,t \\ unskld}} \cdot PL_{\substack{pr,er,t \\ unskld}}$	Value added from labour factor (fa:=l_{skld},l_{unskld})	[113]
$VA_{fa,pr,er,t} = KAV_{pr,er,t} \cdot PK_{pr,er,t} + \sum_{poi} \left((1 - SHAUCTBR_{poi,pr,er,t}) \right) \cdot SALEP_{poi,pr,er,t}$	Value added from labour factor (fa:=k)	[114]
$VA_{fa,pr,er,t} = RESFV_{pr,er,t} \cdot PRESF_{pr,er,t}$	Value added from resources factor (fa:=r)	[115]
$FSEFAT_{fa,pr,er,t} = \sum_{br} VA_{fa,br,er,t}$	Total payment of factors (fa:=l, k, r)	[116]
$FSEFA_{se,fa,er,t} = txstateown_{fa,er,t} \cdot \sum_{br} VA_{fa,br,er,t}$	Factor payments to government (se:=g)	[117]
$FSEFA_{se,fa,er,t} = FSEFAT_{fa,er,t} - \sum_{sr} FSEFA_{se,fa,er,t}$	Labour factor payment to household (se:=h), (fa:=l), (sr≠h)	[118]
$FSEFA_{se,fa,er,t} = FSEFAT_{fa,er,t} - \sum_{sr} FSEFA_{se,fa,er,t}$	Labour factor payment to household (se:=f), (fa:=k,r, (sr≠f)	[119]
$FSEFA_{se,fa,er,t} = txfsefahk_{er,t} - \sum_{br} VA_{fa,br,er,t}$	Capital factor payment to household (se:=h), (fa:=k,r)	[120]
$FSEFA_{se,fa,er,t} = 0$	No labour income transfers to firms (se:=f), (fa:=l)	[121]
$FSEFA_{se,fa,er,t} = 0$	No factor income transfers to world (se:=w)	[122]

where:

$FGRF_{gvf,fa,er,t}$: the payments by factors to public sector expenditure categories,

$txstateown_{fa,er,t}$: the parameter indicating the share of the government to capital income (as calculated in base year),

$\underset{skld}{LAV_{pr,er,t}}$ and $\underset{unskld}{LAV_{pr,er,t}}$: the demand for skilled and unskilled labour in hours,

$\underset{skld}{PL_{pr,er,t}}$ and $\underset{unskld}{PL_{pr,er,t}}$: the price of skilled and unskilled labour,

$KAV_{pr,er,t}$: the capital stock,

$PK_{pr,er,t}$: the user cost of capital,

$RESFV_{pr,er,t}$: volume of reserves

$PRESF_{pr,er,t}$:price of reserves

$FSEFAT_{fa,pr,er,t}$: the total payments by factors,

$SHAUCTBR_{po1,pr,er,t}$: share of auctioned permits per household

$txfsefahk_{er,t}$: the parameter indicating the share of household to capital income (as calculated in base year)

In a general equilibrium context, total savings of a country equal total investments as implied by the Law of Walras.

Final consumption of the sectors of the economy is given in equations below:

$$FC_{se,er,t} = HCDTOT_{er,t} \qquad \text{se=h} \qquad [123]$$

$$FC_{se,er,t} = 0 \qquad \text{se=f} \qquad [124]$$

$$FC_{se,er,t} = \sum_{pr} PGC_{pr,er,t} \cdot GCV_{pr,er,t} \qquad \text{se=g} \qquad [125]$$

$$FC_{se,er,t} = \sum_{pr} \left(PWE_{pr,er,t} \cdot \sum_{eu} EXPO_{pr,er,eu,t} + PWE_{pr,er,t} \cdot YVTWR_{pr,er,t} \right) \qquad \text{se=w} \qquad [126]$$

where:

$PGC_{PR,ER,T}$: the price of delivery to domestic consumption.

The savings of each sector, which if summed up on all economic sectors are equal to total investments, are given below and are computed as the difference between revenues which consists of the receipts from the branches plus income from factors and sectors) and expenditures (which include final consumption and transfers to factors and sectors):

$$SAVE_{se,er,t} = YDISP_{er,t} - HCDTOT_{er,t}$$ se=h [127]

$$SAVE_{se,er,t} = \sum_{fa} FSEFA_{se,sr,er,t} + \sum_{sr} FSESE_{se,sr,er,t} - FC_{se,er,t} - \sum_{sr} FSESE_{sr,se,er,t}$$ se=f [128]

$$SAVE_{se,er,t} = \sum_{gv}\sum_{br} FGRB_{gv,pr,er,t} + \sum_{fa} FSEFA_{se,sr,er,t} + \sum_{sr} FSESE_{se,sr,er,t} - FC_{se,er,t} - \sum_{sr} FSESE_{sr,se,er,t}$$ se=g [129]

$$SAVE_{se,er,t} = \sum_{pr}\sum_{rc} PWXO_{pr,er,cr,t} \cdot IMPO_{pr,er,cr,t} + \sum_{fa} FSEFA_{se,sr,er,t} + \sum_{sr} FSESE_{se,sr,er,t} - FC_{se,er,t} - \sum_{sr} FSESE_{sr,se,er,t}$$ se=w [130]

where :

$YDISP_{er,t}$: Household's disposable income given by equation below

$$YDISP_{er,t} = \sum_{fa} FSEFA_{se,fa,er,t} + \sum_{er} FSESE_{sr,se,er,t} - \sum_{sr} FSESE_{sr,se,er,t}$$ se=h [131]

From the equations described above and the surplus/deficit equation of each sector [132], which is evaluated by subtracting investment and stock variation from gross savings, ensures that total sector savings equal total sector investments (this equality does not hold on a sector level).

$$SURPL_{se,er,t} = SAVE_{se,er,t} - INV_{se,er,t} - TRCAP_{se,er,t}$$ [132]

where:

$SAVE_{se,er,t:}$ the savings by sector,

$INV_{se,er,t:}$ the investments in value,

$TXSTOCKS_{se,er,t}$: the share of sectors in stock variation,

$TRCAP_{se,er,t:}$ the transfer of capital by sector

4.8 Numeraire

In the world version of GEM-E3 numeraire is computed according to the quantitative theory of money, $\mathrm{M} \cdot V = P \cdot Q$ where *M* is money, *V* is the transactions velocity of money, *P* is the price and *Q* the total outlay.

Equation [133] describes total outlays on primary production factors as a function of the base year outlays and the money num. The dual price of this equation determines the worlds' interest rate ($RLTLRWORLD_t$) and consists one of the alternative methods of closure of the model. This equation consists one of the alternative methods of closure of the model and determines the worlds' interest rate (*$RLTLRWORLD_t$*).

$$\sum_{er}\sum_{pr}\left(PK_{pr,er,t}\cdot KAV_{pr,er,t}+PL_{\substack{pr,er,t\\skld}}\cdot LAV_{\substack{pr,er,t\\skld}}+PL_{\substack{pr,er,t\\unskld}}\cdot LAV_{\substack{pr,er,t\\unskld}}+PRESF_{pr,er,t}\cdot RESFV_{pr,er,t}\right)=numvalf_t\cdot priceindex_t\cdot gdp\_growthrate_t \quad [133]$$

Where :

$$numvalf_t=\sum_{er}\sum_{pr}\left(\overline{PK_{pr,er,t}}\cdot\overline{KAV_{pr,er,t}}+\overline{PL_{\substack{pr,er,t\\skld}}}\cdot\overline{LAV_{\substack{pr,er,t\\skld}}}+\overline{PL_{\substack{pr,er,t\\unskld}}}\cdot\overline{LAV_{\substack{pr,er,t\\unskld}}}+\overline{PRESF_{pr,er,t}}\cdot\overline{RESFV_{pr,er,t}}\right)$$

$priceindex_t$: the world price index
$gdp\_growthrate_t$: the worlds' growth rate

4.9 Derived prices – Firms pricing

Derived prices are those depending on leading prices, which are derived from market equilibrium. On derived prices appropriate taxation is applied, to form prices as perceived by consumers. The main leading price is that of the composite good. Depending on the destination of a commodity, differentiated taxation may be applied, as for example indirect taxation or VAT.

4.9.1 Derived prices equations

The prices of goods at intermediate consumption are given in [134], while the prices of goods in final consumption are computed through [135] for households and [136] for government. Finally, [137] defines the prices of goods used to build investment.

$$PIO_{pr,er,t} = txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} + PY_{pr,er,t} \qquad [134]$$

$$PHC_{pr,er,t} = \left(1 + txvat_{pr,er,t}\right) \cdot PY_{pr,er,t} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \qquad [135]$$

where:

$txvat_{pr,er,t}$: the rate of value added tax imposed on good *PR*.

$$PGC_{pr,er,t} = \left(1 + txvat_{pr,er,t}\right) \cdot PY_{pr,er,t} + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \qquad [136]$$

$$PINVP_{pr,er,t} = \begin{cases} \left(1 + txvat_{pr,er,t}\right) \cdot PY_{pr,er,t} + txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} & if\ pr = cns \\ txit_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} + PY_{pr,er,t} & if\ pr \neq cns \end{cases} \qquad [137]$$

The unit cost of investment by sector of destination (owner) depends on its composition in investment goods (by sector of origin). This structure is represented by a set of fixed technical coefficients $tinvpv_{pr,br,er,t}$:

$$PINV_{br,er,t} = \begin{cases} PINV0_{br,er} \cdot \sum_{pr} PINVP_{pr,er,t} \cdot tinvpv_{pr,br,er,t} & if \sum_{pr} tinvpv_{pr,br,er,t} \neq 0 \\ PINV0_{br,er} & if \sum_{pr} tinvpv_{pr,br,er,t} = 0 \end{cases} \qquad [138]$$

4.9.2 Firms pricing

Firms address their products to three market segments namely to the domestic market, to the other EU countries and to the rest of the world. Prices are derived through demand/supply interactions. In any iteration of the model run and before global equilibrium is achieved, producers face demand for their products. To this demand they respond with a price. For the PC sectors, since these operate under constant returns to scale and the number of firms is very large, this price depends only on their marginal cost of production.

The producer is assumed not to differentiate his price according to the market to which he sells his products. He therefore sells his products at the same price (equal to his marginal cost reduced by the amount of production subsidies that he receives).

$$PXD_{brt,er,t} = PD_{brt,er,t} + txsub_{brt,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} + PIMP_{brt,er,t} \cdot rtnc_{brt,er,t} \quad [139]$$

$$PWE_{pr,er,t} = PD_{pr,er,t} + txsub_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} + PIMP_{pr,er,t} \cdot rtnc_{pr,er,t} \quad [140]$$

where:

$PXD_{brt,er,,t}$: the (domestic) supply price addressed to domestic demand,

$PWE_{pr,er,t}$: the (domestic) supply price addressed to exports,

$txsub_{pr,er,t}$: the rate of subsidies

4.10 Equilibrium of the real part

The equilibrium of the real part is achieved simultaneously in the goods market and in the labour market. In the goods market a distinction is made between tradable and non-tradable goods. For the tradable goods the equilibrium condition refers to the equality between the supply of the composite good, related to the Armington equation, and the domestic demand for the composite good. This equilibrium combined with the sales identity, guarantees that total resource and total use in value for each good are identical. For the non-tradable, there is no Armington assumption and the good is homogeneous. The equilibrium condition serves then to determine domestic production.

$$XD_{pr,er,t} = \begin{cases} XXD_{pr,er,t} + \sum_{eu} EXPO_{pr,er,eu,t} \; for\ br = brt \\ Y_{pr,er,t} + \sum_{eu} EXPO_{pr,er,eu,t} + YVTWR_{pr,er,t} - IMP_{pr,er,t} for\ br = brnt \end{cases} \quad [141]$$

Equation [142] describes that the total supply of goods (domestically produced and imported) expended to intermediate consumption, private and public consumption and investments.

$$Y_{pr,er,t} = \sum_{br} \left(IOV_{pr,br,er,t,} + ABIOV_{pr,br,er,t} + INVPV_{pr,br,er,t} + IMAT\_FLOW_{pr,br,er,t} \right) + HCV_{pr,er,t} + GVC_{pr,er,t} + \left(EFFI\_FLOW\_H_{er,t} \cdot nrgeffi\_bcap\_h_{pr,t} \right) \quad [142]$$

In the dual version, this equation determines the total production, the dual price equation gives the production price and the equilibrium condition on the capital market determines the rate of return of capital.

Three alternative choices for the capital mobility are assumed in the model:

i) Capital is immobile between sectors and between regions.

$$KAV_{pr,er,t} = KAVC_{pr,er,t-1} \quad [143]$$

ii) Mobility across sectors but not across regions.

$$\sum_{pr} KAV_{pr,er,t} = \sum_{pr} KAVC_{pr,er,t-1} \quad [144]$$

iii) Full mobility across sectors and regions.

$$\sum_{pr}\sum_{er} KAV_{pr,er,t} = \sum_{pr}\sum_{er} KAVC_{pr,er,t-1} \quad [145]$$

where:

$KAVC_{pr,er,t}$: the total amount of capital stock available, fixed within the time period. Depending on the capital mobility choice, through the switch parameter swonkm(rtime) (i.e. 0 for no mobility, 1 for mobility between sectors, 2 for full mobility and 3 for mobility between specific sectors) , the dual price of the capital $PK_{pr,er,t}$, results from equation[143], [144], [145], as $PKNOKM_{pr,er,t}$, $PKNAKM_{pr,er,t}$ and $PKEUKM_{pr,er,t}$ respectively.

In particular:

$$PK_{pr,er,t} = \begin{cases} PKNOKM_{pr,er,t} \ if \ SWONK = 0 \\ anakm_{pr,er,t} \cdot PKNAKM_{pr,er,t} \cdot XKNUM1_{er,t} \ if \ SWONK = 1 \end{cases} \quad [146]$$

Where $anakm_{pr,er,t}$ is a calibrated parameter and $XKNUM_t$, $XKNUM1_{er,t}$ are used in order to ensure that the computation of $anakm_{pr,er,t}$, $aeukm_{er,t}$, , is consistent with unit cost of capital of sectors both in the baseline and the scenario.

$$XKNUM1_{er,t} = \sum_{pr} \frac{KAV_{pr,er,t}}{\sum_{pr} KAV_{pr,er,t} - anakm_{pr,er,t}} \quad [147]$$

Similarly, $XKNUM1_{er,t}$, $XLNUM_{\underset{skld}{er,t}}$ and $XLNUM_{\underset{unskld}{er,t}}$ are used in order to ensure that the computation of $tl_{\underset{skld}{pr,er,t}}$ and $tl_{\underset{unskld}{pr,er,t}}$ is consistent with unit cost of labour of sectors both in the baseline and the scenario.

$$XLNUM_{\underset{skld}{er,t}} = \frac{\sum_{pr} LAV_{\underset{skld}{pr,er,t}}}{\sum_{pr}\left(\frac{LAV_{\underset{skld}{pr,er,t}} \cdot tl_{\underset{skld}{pr,er,t}}}{(1 + idea_{er,t})}\right)} \quad [134]$$

$$XLNUM_{\underset{unskld}{er,t}} = \frac{\sum_{pr} LAV_{\underset{unskld}{pr,er,t}}}{\sum_{pr}\left(\frac{LAV_{\underset{unskld}{pr,er,t}} \cdot tl_{\underset{unskld}{pr,er,t}}}{(1 + idea_{er,t})}\right)} \quad [148]$$

5 Welfare measure

The quantification of the effects of a policy scenario on GDP, trade, production and the relative prices is done with the computation of the percentage change of the latter from the reference scenario. However the same cannot apply to household welfare where the welfare functions consist ordinal sizes and their summing up (between different households/countries) or the computation of their change from the reference scenario are not possible.

The approach adopted in most of the applied general equilibrium models regards the use of the monetary utility function, which measures the nominal income that the consumer needs for a given price vector in order to be at the same welfare level with a different income level and a price vector. With this measure it is possible to quantify the effects on welfare of alternative policy scenarios.

The specific measure used in the model is that of equivalent variation in welfare given from equation [149] (Robichaud, 2001). This measure shows the income that should be given to/taken off the consumer so as to be found at the same welfare level found with the reference scenario prices. A positive value of this measure means a positive change in consumer welfare. In order to estimate the indirect welfare function (*IU*) we substitute the demand equation $HCDTOTV_{er,t}$ obtained from the maximization program of the household (equation [7]) in the utility function.

$$IU = YDISP - \sum_i \gamma_i \cdot P_i \cdot \prod_i \left(\frac{b_i}{p_i}\right)^{b_i}$$

The definition of equivalent variation:

$$EV = IU\left(P_i^0, v\left(P_i^1, Y_i^0\right)\right) - IU\left(P_i^0, v\left(P_i^0, Y_i^0\right)\right)$$

or equivalently:

$$EV = \left(\frac{IU^n - IU^0}{IU^0}\right) \cdot YDISP^0$$

Thus the exact expression used in the model is:

$$EV = \prod_i \left(\frac{P_i^0}{P_i^1}\right) \cdot \left(YDISP^1 - \sum_i \gamma_i \cdot P_i^1\right) - \left(YDISP^0 - \sum_i \gamma_i \cdot P_i^0\right) \quad [149]$$

6 Construction of the GEM-E3 Reference Case

Constructing a reference case involves the calibration of exogenous variables so as to allow the model to simulate i) a specific regional economic development and ii) policies that induce structural changes to the economy. In GEM-E3 these exogenous variables are: active population, technical progress (capital, labour, energy material) and exogenous expectations on future sectoral growth. The E3M-Lab of the ICCS has developed a methodology and appropriate tools for calibrating GEM-E3 closely to exogenously given values and trajectories, by ensuring that shifts in model exogenous variables are relatively small and within the economic realism.

6.1 Automated baseline methodology.

The methodology of constructing the GEM-E3 baseline respects the logic and structure as well as the dynamic properties of the GEM-E3 model and therefore maintains consistency while allowing for great flexibility on target choices and their hierarchy, as well as calibration instruments. It involves the use of three distinct tools which are encoded in GAMS at the Labouratory's computers: the GEM-E3 model itself, a linearization facility and a Parameter Calibration Model (PCM).

$$G(x \perp s) = 0$$

is the symbolic representation of the entire GEM-E3 model where **G(.)** is the complete set of its equations and **x** an n-dimensional column vector containing all its endogenous variables, while **s** is a vector representing parameters that can be used as instruments, for developing the Baseline. All the key relations of GEM-E3 are homogeneous (CES, Linear Expenditure, Leontief etc) and they are therefore amenable to straightforward and locally accurate linearization when expressed in terms of differences:

$$A \cdot \dot{x} + B \cdot \dot{s} = b$$

where

$$\dot{x} = \frac{dx}{dt}, \dot{s} = \frac{ds}{dt}$$

A (n x n) and B (n x m) are coefficient matrices,

s (m x 1) a vector of control variables (m < n)

b is normally an (n x 1) vector of zeros

The control variables **s** represent parameters of the GEM-E3 Model such as embodied and disembodied factor productivities, habit parameters in demand functions, exogenous parameters on resource availability (e.g. active population), income distribution parameters, risk premium parameters, structural shifts in technology or other input uptake, other parameters which normally define inequality constraints.

The linearized model is incorporated in a goal programming model which solves the following linear program:

$$min_{x,s} \ \ w'(u_1 + u_2)$$

$$s.t. \ A\dot{x} + B\dot{s} = b$$

$$\dot{x} + u_1 - u_2 = \dot{y}$$

$$u_1 \geq 0, u_2 \geq 0, \dot{x} \in \Re^n, z_\ell \ \leq \dot{s} \leq z_\hbar$$

Where:

y	is an n x 1 vector of target variables and
w	is an n x 1 vector mapping the importance of getting close to growth for a given target variable
$\boldsymbol{z_l}$ and $\boldsymbol{z_h}$ are vectors of lower and upper bounds defining permissible ranges for the control variables (parameters of the model)

The target variables are user-defined. Those that are of no interest as targets can feature with a w value equal to zero. They must be sufficiently numerous to avoid dual degeneracy (multiple optima). They must also be sufficiently few to avoid having to pre-determine too many values.

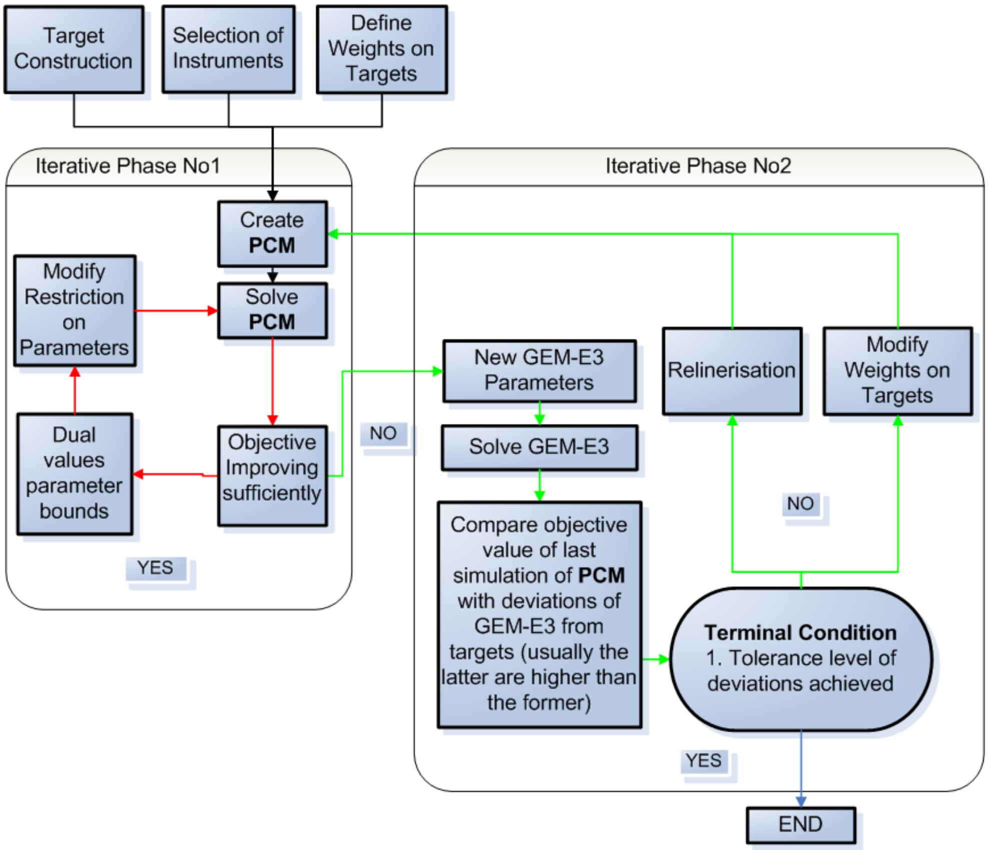
The linear program above constitutes the core of the Parameter Calibration Model (PCM).The main advantage of using a linear programming formulation for the PCM is the very high speed with which, even very large such programmes, can be solved by modern computers. This is particularly important in view of the fact that the calibration procedure involves many successive runs of this model. The overall procedure is described in Figure 12. Having established the values of the target variables, decided on the instruments to be used in meeting them (as well as any restrictions concerning them) and deciding on their relative importance, the procedure involves two iteration phases which are performed successively.

In the first phase, after linearising GEM-E3 around its current solution (not meeting the targets), the linearised model is incorporated in the PCM which is then solved successively by modifying the bounds on the parameters that are used as instruments.

In choosing the extent of relaxation of the different bounds guidance is provided by the dual values as obtained from the previous run of the PCM.

In this way the relaxation proceeds on a wide front involving many instruments but at the same time it is selective in that the relaxation is more extensive for those instruments that display the greater promise in terms of improving the objective function. In general the improvements in the latter become less pronounced with successive runs of the PCM and a point soon is reached that a re-construction of the PCM becomes necessary. This is prepared within iterative phase 2. Here the last solution of the PCM is introduced into GEM-E3 and the "true" (non-linear) version of the model is solved. In general, deviations from targets will be higher than suggested by the last solution of the PCM because the linearised version only constitutes an approximation of the "true" model at a solution point that has by now been superseded. Unless deviations are acceptably small the PCM must be re-constructed by re-linearising and increasing the importance of targets displaying large deviations. The procedure subsequently moves back into iterative phase one. The two phases are executed successively until an overall satisfactory result is obtained (acceptable deviations). The whole procedure must be repeated separately and successively for every solution year of the model.

Figure 12:Automated baseline procedure diagram.



7 Calibration

The standardised version of the problem of model parameter estimation is formulated as follows: Taking into account a system of equations *F(Y, X,b,e)=0* where *Y* is a vector of the n endogenous variables , *X* is a vector of the exogenous variables, *b* is a vector of unknown parameters and e is a vector of non-stochastic residuals of a known or an unknown distribution, the problem consists of defining the values of the vector *b* such that *e* takes the smallest possible value.

The literature provides three alternative answers to this question:

- Econometric estimation (Brundy and Jorgenson, 1974, Jorgenson and Laffont, 1974, Jorgenson, 1984)
- Adjustment to the base year- Calibration, (Mansur and Whalley, 1984)
- Employment of entropy methods (Robinson et al, 1998)

The method which is widely used is that of the adjustment to the base year (calibration). This method regards setting the components of e equal to 0 and solving for vector b based on single observations of Y and X. However to the degree that b has more than n components (i.e. m-n), more information is required so as to determine the number of the m-n unknown parameters. Consequently the method of base year adjustment adopts a strong assumption that the observed values of the endogenous variables are set only from the factors included explicitly in the model. A common practice in this method is for some of the parameters to be set based on the relevant literature; in this way although some of the parameters are chosen arbitrarily the rest of them take the values necessary for the model to reproduce the base year data.

The main critics of this approach are Jorgenson (1984), Lau (1984), Diewert and Lawrence (1994) and McKitrick (1998). Their critique can be summarised as follows: Researchers often use elasticities which are calculated for product classification, which are not completely in accordance with those employed in the model or for countries which are not represented by the model. The method of adjustment to the base year (calibration) forces the quality of the model to depend, at least partially, on the quality of the data of an arbitrarily chosen base year. Jorgenson (1984) argues that: “the choice of a single base year means that whatever stochastic irregularities are present in the observations for that period will inevitably affect the structure of the model. The parameters extracted from the literature may be outdated or refer to different industries, products and geographic regions than those set in the model”.

In order to keep the number of the parameters to be estimated quite low, the representation of the preferences and of the technology should be based to a large extent on CES or Cobb-Douglas functions, i.e. functional forms with a small number of parameters. This implies that very restrictive assumptions on preferences and technology should be accepted. Despite the disadvantages the method of adjustment to the base year is predominant. This is associated with a very important feature of this method: the adequacy of few data. Indeed few countries in the world can provide social accounting matrices[17] for

[17] Square matrices which describe all the economic transactions of an economic system at a given time.

long time series. Hence, when constructing a general equilibrium model containing a large number of countries/regions, the adoption of this method is the only solution.

With regards to the restrictive assumptions set on consumer and producer preferences from the choice of relatively simple consumption and production functions, it could be argued that if sensitivity analysis was made on the model regarding the values of the independent unknown parameters, the results could be interpreted and understood more easily due to the small number of the parameters but also due to the simple functional forms used.

The obvious way to overcome these constraints and to increase the empirical relevance of the CGE models is the econometric estimation of the parameters. Nevertheless there are significant difficulties associated with such an approach. More specifically the dimensions of the applied models cause serious degrees of freedom problems, especially if the constraining assumptions on the structure of the preferences and on technologies are avoided. Moreover the simultaneous estimations of a general equilibrium model require quite complicated econometric techniques (Lau, 1984; Whalley and Mansur, 1984).

Another way of parameter estimation is the use of cross entropy. This method is the extension of the method of direct adjustment to the base year in the sense that it can take advantage of statistics coming from various sources and years. The theory of this technical estimation is described in S. Robinson et al (2001).

7.1 The choice of functional forms in general equilibrium models

The choice of the functional form which will be used in an applied model is constrained by the goals of the research, the available data and the theoretical context of the model. Following these, three criteria are usually applied:

- Small number of equation parameters (function should not have other parameters than the required)
- Ease of interpretation (the usual functional forms do not have intuitive economic interpretation)
- Computational ease

In the case of CGE models the functions used widely are those of constant elasticity of substitution (CES). CES is a generalized form of the Cobb-Douglas function (special case where the elasticity of substitution equals 1) and the Leontief constant coefficient function (special case where elasticity of substitutions tends to 0). CES properties agree with the standard requirements of general equilibrium. More specifically, this function is defined for positive input levels, is continuous, differentiable, monotonic (an increase in inputs cannot reduce production), strictly concave and homogenous of degree 1 (constant returns to scale). In addition it is appropriate for the application of the Euler theorem and has homogenous average and marginal product of degree 0.

Most versions of the CES function can be considered the result of the attempts to overcome the assumption employed in the multifactor form of CES, i.e. equality of all the partial Allen-Uzawa elasticities (Uzawa, 1962; McFadden, 1963). An extension which relaxes this constraint is the hierarchically structured CES function (Sato, 1967). The basic disadvantage of the aggregate multi-variable functions is that they do not represent the technological conditions that a firm faces according to the "production function" term but they consist of a theoretical form which approximates them.

The first step for running the calibration procedure[18] of the GEM-E3 model, is to define values for the elasticities that determine all coefficients that do not correspond to directly observable variables and then to run the calibration procedure. The calibration module is written as a separate model and has a recursive structure. The base-year data, used for calibration, correspond to monetary terms, therefore appropriate price indices are chosen to compute the corresponding volumes (quantities). The present version of the model uses values of elasticities from the literature or guess-estimated when no econometric estimates are available. The calibration procedure requires data for a single year, which is considered as the base year of simulation. Data for a year previous to the base-year are required to give values to those variables that are lagged in the model. The calibration procedure is defined in such a manner that the model reproduces exactly the observed statistics of the base year.

Three main sets of elasticities are used in the GEM-E3 model:

- Demand function elasticities following the Armington assumption adopted in the model (substitutability of domestic/imported goods and across imported goods, by country of origin).
- Elasticities of substitution in production (substitution among production factors).
- Consumer preferences (price or income elasticities in households demand for commodities).

7.2 Elasticities

7.2.1 Armington elasticities

Despite of the popularity of the Armington concept, only few studies on direct econometric estimates of substitution elasticities have been published. Elasticities of upper-level substitution between imported and domestic goods have been estimated, for example, by Reinert and Roland-Holst (1992), Shiells et al. (1986) and Lächler (1985). Shiells and Reinert (1993) have estimated lower-level elasticities and non-nested elasticities, as well as Sobarzo (1994), and Roland-Holst et al (1994). Unfortunately, the estimated values from the literature are difficult to compare, as the sectoral aggregation levels differ according to the statistical data base used.

A study for Germany was conducted by Lächler (1985). Lächler estimated disaggregated elasticities of substitution between demand for imports and domestic substitutes in Germany. He found that the primary goods industry which consists of relatively homogeneous and easily replaceable goods and which is under high pressure in terms of international competitiveness is the one with the highest elasticity ranking: Apart from two exceptions, elasticity values range from 0.233 to 2.251. In contrast, in the case of the

[18] The file that starts the calibration procedure is AA_Calibmain.gms

investment goods sector, and particularly in the case of capital goods in the short run, technological rigidities restrict the substitutability; thus, elasticity values are rather low and between the range of -2.283 to 1.209. Finally, the sectors that are classified as belonging to the consumption goods industry differ with respect to the degree of international competitive pressure, reflected by wide differences in measured substitution elasticities (-0.697 to 1.092).

Likewise, Reinert and Roland-Holst (1992) have estimated elasticities of substitution between imported and domestic goods for 163 U.S. mining and manufacturing sectors, based on U.S. trade data time series of both prices of domestic and imported goods, and real values of domestic sales of domestic goods and imports. In about two-thirds of the cases Reinert and Roland-Holst obtained positive and statistically significant estimates ranging from 0.14 to 3.49. Their results allow the conclusion that at the level of aggregation chosen imports and U.S. domestic products are far away from being perfect substitutes.

Furthermore, Shiells et al. (1986) have published estimations on disaggregated own-price elasticities of import demand for 122 3-digit SIC U.S. industries (covering mainly mining and manufacturing sectors) which serve as a basis for inferring upper-level substitution elasticities. The estimations are based on annual data for period 1962-1978. In 48 cases positive and statistically significant elasticities of substitution were obtained, ranging from 0.454 for SIC 208 (beverages) to 32.132 for SIC 373 (yachts).

Shiells and Reinert (1993) estimated both lower-level nested and non-nested elasticities of substitution among U.S. imports from Mexico, Canada, RoW, and competing domestic production, for 22 mining and manufacturing sectors, based on quarterly data for 1980-88. In the non-nested specification, U.S. imports from Mexico, Canada, and RoW as well as domestic substitutes enter a single CES function. The estimates of the non-nested elasticities of substitution range from 0.101 (sector primary lead, zink, and non-ferrous metals, n.e.c.) to 1.49 (sector primary aluminium). The nested specification is composed of an upper-level CES aggregation function for U.S. imports as a whole and a lower-level CES aggregate function for the various import sources, i.e. lower-level substitution elasticities are among U.S. imports from Mexico, Canada, and RoW. Estimates range from 0.04 (sector clay, ceramic, and non-metallic minerals) to 2.97 (sector iron, and ferroalloy ores mining).

A comparison of estimates for non-nested, lower-level and upper-level elasticities for selected sectors taken from Shiells and Reinert (1993) and Reinert and Roland-Holst (1992) show that values differ. While the non-nested estimates lie mainly above the upper-level estimates, they are in half of the cases lower and in half of the cases higher than the lower-level estimates. As already mentioned in Section 7.1, lower-level elasticities are not generally higher than upper-level elasticities, but only in about two thirds of the sectoral cases considered in the table. However, lower-level estimates show that the range of positive values (0.04 - 2.97) is larger, as in the case of the non-nested specification (0.1 - 1.49) and in the case of upper-tier estimates (0.02 - 1.22).

Gallaway et al (2002) provides short-run and long-run industry-level estimates of U.S. Armington elasticities based on high frequency monthly data for 309 manufacturing industries at the four-digit Standard Industrial Classification (SIC) level over the period 1989–1995. They found that on average, long-run estimates are approximately two times larger than the short-run estimates. The highest short run elasticities were estimated for the metals sectors (2.7 on average). The GTAP(2006) database is a source of trade elasticities at two levels: i) Domestic/Imported and ii) between different countries. These elasticities are provided for each commodity included in the GTAP database.

Annex V contains the upper-level and the lower-level Armington elasticity values actually used in the GEM-E3 model. Elasticities differ among sectors, but values for each sector are identical for all countries/regions. It also contains income elasticities per consumption categories for the 38 countries of the model.

Non-tradable sectors and non-competitive imports are treated in a different way. Both import demand of non-traded and non-competitive commodities are excluded from the Armington assumption. It is assumed that they are determined not by price relations but by the domestic production level and institutional settings, such as supply contracts. More importance should be attached to the problem arising from non-competitive imports. Given the same import price elasticity value, the share of non-competitive imports assumed influences the inferred Armington elasticity values. It can be stated that the higher the share of non-competitive imports, the higher the Armington elasticity which corresponds to a given import price elasticity. In the GEM-E3 model the shares of non-competitive imports are set equal to 0.5 for all countries and all sectors.

7.2.2 Elasticities of substitution in production

Many econometric studies have attempted to estimate the substitution possibilities between the production factors within an integrated production model. They point out to the importance of the number of productions factors specified and of the specification of technical progress. The distinction between electricity and other fuels is necessary because the substitution mechanism and possibilities between these energy factors and the other production factors are different. The specification of the technical progress has a clear impact on the estimated substitution elasticities.

A review of the literature on the estimation of elasticities of capital to labour substitution reveals a somewhat confusing array of results. Nerlove (1967) concludes that even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity. Zarembka (1970) challenges this view and argues that a correction of the labour and wage-rate variables for quality variations and the use of seemingly unrelated regression lead to results such that the use of different time periods does not produce different estimates of the elasticity. Griliches (1967) concludes that labour-quality variables contribute little to the estimation of the elasticity.

Berndt (1976) lists a variety of hypotheses that have been advanced to explain the diversity of results but concludes that, in general, empirical studies attempting to take account of these deficiencies have produced unsatisfactory results. Morawetz (1976) finds similar results after examining several studies for several developing countries[19].

[19] He noted that it was impossible to find industries with consistently high or low elasticities in either developing countries or advanced economies, such as the United States of America

Comprehensive reviews of empirical studies are given by Walters (1963), Nerlove (1967) and Gaude (1975).

It is generally observed that the elasticity estimates obtained from time-series data are significantly lower than those obtained from cross-sectional data. Boddy (1967) attributes this results to the fact that the variability of data used for time-series estimates is limited compared to cross-sectional samples. Gaude (1975) supports that the lower estimates obtained from time-series data are due to the simultaneity between inputs and their prices, misspecification of adjustment lags between inputs and outputs and the dominance of cyclical conditions in time-series data.

Attempts of estimating the elasticity of substitution have been the focus of several empirical studies. Arrow et al (1961) and Berndt (1976) among others have developed a set of influential works. The idea is to estimate the elasticity of substitution directly from cost minimizing 1st order conditions with respect to the factors of production, assuming competitive product and factor markets and a CES production technology. This approach has recently been employed in times series studies by Balisteri et al. (2003), Klump and De La Grandville (2000) and Antras (2004), among others.

Several empirical limitations impede this approach. These include the possibility of biased technological growth and endogenous regressors. In addition Antras (2004) and Jalava et al (2006) recognize that the typical data used to estimate the elasticity display non-stationary, trending behaviour. The approach suggested in the work of Arrow et al (1961) and Berndt (1976) is applicable when the underlying assumptions on competition and technological change are approximately valid, provided that the main characteristics of the data, such as stochastic trending, are carefully handled. Berndt (1976) and Antras (2004) found that the estimates of the elasticity based on the marginal product of labour equations tend to be higher than the estimates based on the marginal product of capital equations.

Juselius (2008) proposes a different approach to drawing inference on the elasticity of substitution, which is based on the idea that estimates of the elasticity of substitution may be retrievable from behavioural equations derived from more realistic models by conducting comparative statics with respect to this parameter. The empirical problem in this case is to investigate if the observed long-run behaviour of the data is consistent with the results of the comparative statics. This approach avoids difficulties with direct inference about elasticity values in a CES production function by exploiting theoretical relationships discovered using an economic model which assumes market imperfections. The drawback of this approach is that no point estimates of the elasticity can be obtained but its merit consists of taking into account the nonstationarity features of the data while remaining closely connected to economic theory (Hassler, 2008).

Initially the nested CES structure and the substitution elasticities in GEM-E3 were based on the econometric study by CES and the Belgian Planning Office on the substitution possibilities in 10 Belgian industrial sectors, as this study was available and took into account the main findings on the specification needed for the modelling of factor demand. In a next version Koschel, Henrike (2000) has estimated substitution elasticities between capital, labour, material, electricity and fossil fuels in Germany for the sectoral classification of the GEM-E3 model.

More recently the WIOD database has been used in order econometrically estimate key parameters of the GEM-E3 model Fragkiadakis C. et al (2012). The aim was to establish econometrically some benchmark values for the constant elasticities of substitution that characterise Computable General Equilibrium models and constitute important elements in controlling their simulation properties. A time series analysis was performed in order to examine the non-stationarity and the autocorrelation of the data series and identify possible long-run equilibrium relations (cointegration). Two estimation methods were used: i) OLS method applied to the first differences of the demand functions (these functions are derived from firms profit maximization) ii) An error correction model applied when a cointegration relationship exists.

The long run point estimates span a range from 0.4489 to 2.8750 for the various sectors of activity and regions. The highest short run elasticities among regions were found in China, India, and Japan, whereas the highest long run elasticities were found in EU15. These results suggest that capital and labour are relatively easily substitutable in these regions both in short term and long term.

Fragkiadakis et al (2012) estimates are consistent with previous empirical results published by Berndt (1976) and by Antras (2004). In fact, the elasticity values based on the marginal product of labour equations tend to be higher than the values based on the marginal product of capital equations. Labour supply elasticities for skilled and unskilled labour and other elasticities used in the GEM-E3 model are presented in Annex V.

7.3 Calibration of LES

At the upper level of the consumption function, household decides between the consumption of goods and savings. It is assumed that households have a minimum level of consumption (chv), this minimum consumption is calibrated to the demand function derived from the maximization problem of the household. The minimum consumption is calculated on the social time preference relative to the interest rate(str) and the disposable income (Ydisp).

$$chv = \frac{HCDTOT - Ydisp \cdot str}{1 - str}$$

str is the social time preference links the upper level of consumption with the lower level and is calibrated based on the following formula:

$$str = \frac{HCDTOT - \sum_{nlnd} chcfv_{nlnd} \cdot pHCFV_{nlnd} - \sum_{dg,lnd} shinv_{dg} \cdot mincons_{lnd,dg} \cdot pHCFV_{lnd} - \sum_{dg} decl_{dg} \cdot chcfv_{dg} \cdot pHCFV_{dg}}{Ydisp - \sum_{nlnd} chcfv_{nlnd} \cdot pHCFV_{nlnd} - \sum_{dg,lnd} shinv_{dg} \cdot mincons_{lnd,dg} \cdot pHCFV_{lnd} - \sum_{dg} decl_{dg} \cdot chcfv_{dg} \cdot pHCFV_{dg}}$$

We use the Harberger convention to get $P_{\mathrm{i}}^{\mathrm{cv}}=1, \mathrm{CV_i}=\frac{\mathrm{chv_i}}{P_{\mathrm{i}}^{\mathrm{cv}}}$. The average budget shares are $ABS_i=\frac{P_{\mathrm{i}}^{\mathrm{cv}}\cdot \mathrm{CV_i}}{\sum_{\mathrm{j}} P_{\mathrm{j}}^{\mathrm{cv}}\cdot \mathrm{CV_j}}$. In order to compute the marginal expenditure shares β_H we use the income elasticity $\varepsilon_m=\frac{\frac{\mathrm{dCV}}{\mathrm{CV}}}{\frac{\mathrm{dI}}{\mathrm{I}}}=\frac{\mathrm{dCV}}{\mathrm{dI}}\cdot\frac{I}{\mathrm{CV_i}}=\frac{\beta_H}{P_i^{cv}}\cdot\frac{I}{\mathrm{CV_i}}=\beta_H\cdot\frac{I}{P_i^{cv}\cdot \mathrm{CV_i}}=\beta_H\cdot(ABS_i)^{-1}$, hence $\beta_H=\varepsilon_m\cdot ABS_i$.

From Engel aggregation we know that the sum of income elasticities weighted by the consumption shares equals 1. This result is easily obtained if we take $\sum_i P_i\cdot X_i=I\rightarrow 1=\sum_i P_i\cdot\frac{X_i}{I}\cdot\frac{dX_i}{dI}\cdot\frac{I}{X_i}\rightarrow 1=\varepsilon_m\cdot ABS_i$. Hence β_i equals always one. To calculate the lower level minimum obliged consumption the Frisch parameter is used. The Frisch Φ parameter presents the marginal utility of income with respect to income and is used as a tool in order to calibrate the household consumption. Frisch parameter is calculated based on the gdp per capita. Its range is between -3.5 and -1.8 and the higher the value of the Frisch parameter is, the lower the obliged consumption is. In the case where the alternative approach for household consumption is used (i.e. the addition of leisure), the utility function at the upper level is a Cobb-Douglas and includes, consumption and minimum consumption, consumption of leisure and minimum consumption of leisure. In this alternative formulation households must decide between two goods (consumption and leisure) and thus a utility maximization should be followed. The maximization problem subject to the budget constraint is as follows:

$$\max_{\mathrm{cv,ljv}} U(CV,LJV)=\left(\beta_H\cdot ln(CV-CH)+\beta_L\cdot ln(LJV-CL)\right)$$
$$\mathrm{s.t.\,I}=\mathrm{P^{cv}}\cdot\mathrm{CV}+\mathrm{P^{ljv}}\cdot\mathrm{LJV}$$

the Lagrange function is :

$$\mathfrak{I}=\left(\beta_H\cdot ln(CV-CH)+\beta_L\cdot ln(LJV-CL)\right)-\lambda\cdot(\mathrm{I}-\mathrm{P^{cv}}\cdot\mathrm{CV}-\mathrm{P^{ljv}}\cdot\mathrm{LJV})$$

$$\frac{d\,\mathfrak{I}}{dCV}=\frac{\beta_H}{CV-CH}-\mathrm{P^{cv}}\cdot\lambda=0$$
$$\frac{d\,\mathfrak{I}}{dLJV}=\frac{\beta_L}{LJV-CL}-\mathrm{P^{ljv}}\cdot\lambda=0$$
$$\frac{d\,\mathfrak{I}}{d\lambda}=\mathrm{I}-\mathrm{P^{cv}}\cdot\mathrm{CV}-\mathrm{P^{ljv}}\cdot\mathrm{LJV}$$
$$CV=\frac{\mathrm{P^{ljv}}}{\mathrm{P^{cv}}}\cdot\frac{\beta_H}{\beta_L}\cdot(LJV-CL)+CH$$
$$LJV=\frac{\mathrm{P^{cv}}}{\mathrm{P^{ljv}}}\cdot\frac{\beta_L}{\beta_H}\cdot(CV-CH)+CL$$

Replacing *cv,ljv* we get

$$I-\mathrm{P^{ljv}}\cdot CL=\mathrm{P^{cv}}\cdot CV+\frac{\mathrm{P^{cv}}\cdot CV}{\beta_H}+\frac{\mathrm{P^{cv}}\cdot CH}{\beta_H}$$

Multiply both sides with $\frac{CH}{\mathrm{P^{cv}}}$ and solve for *CV* and *LJV*

$$CV^*=CH+\frac{\beta_H}{\mathrm{P^{cv}}}\cdot\left(I-\mathrm{P^{cv}}\cdot\mathrm{CH}+\mathrm{P^{ljv}}\cdot CL\right)$$
$$LJV^*=CL+\frac{\beta_L}{\mathrm{P^{ljv}}}\cdot\left(I-\mathrm{P^{cv}}\cdot\mathrm{CH}+\mathrm{P^{ljv}}\cdot CL\right)$$

7.4 Calibration of the efficiency wage function

Empirical evidence (Blanchflower and Oswald (1994), Galdeano and Turunen (2005)) show that the unemployment elasticity of real wages does not vary across counties and is found to be close to -0.1. In the efficiency wage curve the parameters *effort* (disutility from working) and *pcaught* (probability to be caught shirking) are computed so that the unemployment elasticity for each country is close to -0.1. Thus the calibration of *pquit* for skilled and unskilled labour(probability of quitting from the job – set exogenously) is:

$$pquit_{\substack{er,t\\skld}} = UNRT_{\substack{er,t\\skld}} \cdot \left[\left(WRMEAN_{\substack{er,t\\skld}} - unben_{\substack{er,t\\skld}} - effort_{\substack{er,t\\skld}} \right) \frac{effort_{\substack{er,t\\skld}}}{pcaught_{\substack{er,t\\skld}}} - stp_{er,t} \right]$$

$$pquit_{\substack{er,t\\unskld}} = UNRT_{\substack{er,t\\unskld}} \cdot \left[\left(WRMEAN_{\substack{er,t\\unskld}} - unben_{\substack{er,t\\unskld}} - effort_{\substack{er,t\\unskld}} \right) \frac{effort_{\substack{er,t\\unskld}}}{pcaught_{\substack{er,t\\unskld}}} - stp_{er,t} \right]$$

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Annex I Definition of National Accounting Concepts

This Annex gives an algebraic representation of the national accounts described in Section 0.

GDP

Income Approach:

$$GDP = \sum_{SS} D1(received) + \sum_{SS} B2G\_B3G(received) + \sum_{SS} D2(received) - \sum_{SS} D3(paid) - \sum_{S2} D1(paid)$$

GDP = + Compensation of employees received by all sectors
+ Gross operating surplus and gross mixed income received by all sectors
+ Taxes on production and imports received by all sectors
- Subsidies paid by all sectors
- Compensation of employees paid by rest of the world

Expenditure Approach:

$$GDP = \sum_{SS} P3(paid) + \sum_{SS} P5(paid) + \sum_{S2} P6(paid) - \sum_{S2} P7(received)$$

GDP = + Final consumption expenditure paid by all sectors
+ Gross capital formation paid by all sectors
+ Exports of goods and services paid by rest of the world
- Imports of goods and services received by rest of the world

Output Approach:

$$GDP = \sum_{SS} B1G(paid) + \sum_{SS} D21(paid) - \sum_{SS} D31(received)$$

GDP = + Gross value added (at basic prices) paid by all sectors
+ Taxes on products paid by all sectors
- Subsidies on products received by all sectors

Gross value Added (at basic prices) (B1G):

$$B1G = \sum_{SS} D1(received) - \sum_{S2} D1(paid) + \sum_{SS} B2G\_B3G(received) + \sum_{SS} D29(received) - \sum_{SS} D39(paid)$$

B1G = + Compensation of employees received by all sectors
- Compensation of employees paid by rest of the world
+ Gross operating surplus and gross mixed income received by all sectors
+ Other taxes on production received by all sectors
- Other Subsidies on production paid by all sectors

Gross national income/Balance of primary incomes (B5G):

$$B5G = GDP - \left(\sum_{S2} D1(received - paid) + \sum_{S2} D2(received - paid) + \sum_{S2} D3(received - paid) + \sum_{S2} D4(received - paid)\right)$$

B5G = + Gross Domestic Production
- Surplus or deficit of the rest of the world on Compensation of employees
- Surplus or deficit of the rest of the world on Taxes on production and imports
- Surplus or deficit of the rest of the world on Subsidies
- Surplus or deficit of the rest of the world on Property income

Gross disposable income (B6G):

$$B6G = B5G - \left(\sum_{S2} D5(received - paid) + \sum_{S2} D6(received - paid) + \sum_{S2} D7(received - paid)\right)$$

B5G = + Gross national income
- Surplus or deficit of the rest of the world on Current taxes on income, wealth, etc.
- Surplus or deficit of the rest of the world on Social contributions and benefits
- Surplus or deficit of the rest of the world on other current transfers

Gross Saving (B8G):

$$B8G = B6G - \sum_{SS} P3(paid)$$

B8G = + Gross disposable income
- Final consumption expenditure paid by all sectors

Gross disposable income (B6G) for each sector:

S11	Non-financial corporations
S12	Financial corporations
S13	General government
S14_S15	Households; non-profit institutions serving households

$$B6G = \sum_{S1J} B2G\_B3G(received) + \sum_{S1J} D1(received) + \sum_{S1J} D2(received) - \sum_{S1J} D3(paid) + \sum_{S1J} D4(received - paid) + \sum_{S1J} D5(received - paid) + \sum_{S1J} D61(received - paid) + \sum_{S1J} D62(received - paid) + \sum_{S1J} D7(received - paid)$$

B6G = + Gross operating surplus and gross mixed income received
+ Compensation of employees received
+ Taxes on production and imports received
- Subsidies paid
+ Property income received
+ Surplus or deficit of the sector on Current taxes on income, wealth, etc.
+ Surplus or deficit of the sector on Social contributions
+ Surplus or deficit of the sector on Social benefits other than social transfers in kind
+ Surplus or deficit of the sector on other current transfers

S2	Rest of the world

$$B6G = \sum_{S2} B2G\_B3G(received) + \sum_{S2} D1(received - paid) - \sum_{S2} D3(paid) + \sum_{S2} D4(received - paid) + \sum_{S2} D2(received) + \sum_{S2} D5(received - paid) + \sum_{S2} D61(received - paid) + \sum_{S2} D62(received - paid) + \sum_{S2} D7(received - paid) + \sum_{S2} D9(received - paid) + \sum_{S2} B11(received) - \sum_{S2} B101(paid)$$

B6G = + Gross operating surplus and gross mixed income received
+ Surplus or deficit of the sector on Compensation of employees.
+ Taxes on production and imports received
- Subsidies paid
+ Property income received
+ Surplus or deficit of the sector on Current taxes on income, wealth, etc.
+ Surplus or deficit of the sector on Social contributions
+ Surplus or deficit of the sector on Social benefits other than social transfers in kind
+ Surplus or deficit of the sector on other current transfers
+ Surplus or deficit of the sector on Capital transfers
+ External balance of goods and services
- Changes in net worth due to saving and capital transfers

Gross savings (B8G) for each sector:

S11	Non-financial corporations
S12	Financial corporations
S13	General government
S14_S15	Households; non-profit institutions serving households
S2	Rest of the world

$$B8G = B6G + \sum_{SJ} D8(received - paid) - \sum_{SJ} P3(paid)$$

B8G = + Gross disposable income received
+ Surplus or deficit of the sector on Adjustment for the change in net equity of households in pension funds reserves
- Final consumption expenditure paid

Investment for each sector:

S11	Non-financial corporations
S12	Financial corporations
S13	General government
S14_S15	Households; non-profit institutions serving households

$$INV = P5 + \sum_{SJ} K2(paid) - \sum_{SJ} D9(received - paid)$$

INV = + Gross capital formation

+ Acquisitions less disposals of non-financial non-produced assets
- Surplus or deficit of the sector on Capital transfers

S2	Rest of the world

$$INV = P5 + \sum_{S2} K2(paid) - \sum_{S2} B101(received)$$

INV = + Gross capital formation
+ Acquisitions less disposals of non-financial non-produced assets
- Changes in net worth due to saving and capital transfers

Net Lending for each sector:

S11	Non-financial corporations
S12	Financial corporations
S13	General government
S14_S15	Households; non-profit institutions serving households
S2	Rest of the world

$$\mathrm{B9} = \mathrm{INV} - \mathrm{B8G}$$

B9 = + Investment
- Gross Savings

Annex II List of regions of the GEM-E3 model

Abbreviation	Country	Abbreviation	Country
AUT	Austria	LVA	Latvia
BEL	Belgium	MLT	Malta
BGR	Bulgaria	NLD	Netherlands
CRO	Croatia	POL	Poland
CYP	Cyprus	PRT	Portugal
CZE	Czech Republic	SVK	Slovakia
DEU	Germany	SVN	Slovenia
DNK	Denmark	SWE	Sweden
ESP	Spain	ROU	Romania
EST	Estonia	USA	USA
FIN	Finland	JPN	Japan
FRA	France	CAN	Canada
GBR	United Kingdom	BRA	Brazil
GRC	Greece	CHN	China
HUN	Hungary	IND	India
IRL	Ireland	AUZ	Oceania
ITA	Italy	FSU	Russian federation
LTU	Lithuania	ANI	Rest of Annex I
LUX	Luxembourg	ROW	Rest of the World

Annex III List of activities of the GEM-E3 model

No.	Activity	No.	Activity
1	Agriculture	Power generation	
2	Coal	22	Coal fired
3	Crude Oil	23	Oil fired
4	Oil	24	Gas fired
5	Gas	25	Nuclear
6	Electricity supply	26	Biomass
7	Ferrous Metals	27	Hydro electric
8	Non-ferrous metals	28	Wind
9	Chemical Products	29	PV
10	Non-metallic minerals	30	CCS coal
11	Paper products	31	CCS Gas
12	Electric Goods		
13	Transport equipment		
14	Other Equipment Goods		
15	Consumer Goods Industries		
16	Construction		
17	Transport (Air)		
18	Transport(Land)		
19	Transport (Water)		
20	Market Services		
21	Non Market Services		

Annex IV Base Year Energy Efficiency Level

Table 7: Base year energy efficiency level

No.	Country/Region	Efficiency level	No.	Country/Region	Efficiency level
1	China	0.13	19	Italy	0.12
2	Japan	0.11	20	Latvia	0.18
3	India	0.25	21	Lithuania	0.17
4	Canada	0.12	22	Luxembourg	0.11
5	USA	0.11	23	Malta	0.12
6	Brazil	0.16	24	Netherlands	0.14
7	Austria	0.11	25	Poland	0.15
8	Belgium	0.12	26	Portugal	0.13
9	Cyprus	0.11	27	Slovakia	0.16
10	Czech Republic	0.17	28	Slovenia	0.13
11	Denmark	0.11	29	Spain	0.11
12	Estonia	0.22	30	Sweden	0.11
13	Finland	0.13	31	United Kingdom	0.10
14	France	0.10	32	Bulgaria	0.23
15	Germany	0.11	33	Romania	0.17
16	Greece	0.13	34	Oceania	0.11
17	Hungary	0.14	35	Russian Federation	0.22
18	Ireland	0.11	36	Rest of Annex I	0.12
			37	Rest of the World	0.20

Annex V GEM-E3 Elasticities

Table 8: GEM-E3 elasticities

	Agriculture	Coal	Crude Oil	Oil	Gas	Electricity supply	Ferrous metals	Non ferrous metals	Chemical Products	Paper Products	Non metallic minerals
	1	2	3	4	5	6	7	8	9	10	11
sn1:Elasticity of substitution between KLE and MA	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
sn2: Elasticity of substitution between KL and ENG(non energy sectors)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
sn3:Elasticity of substitution between intermediate goods	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
sn4:Elasticity of substitution between K and skilled and unskilled L	0.23	0.20	0.20	1.26	0.73	1.26	1.26	1.26	1.26	1.26	0.73
sn5:Elasticity of substitution between Energy and Electricity	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
sn6:Elasticity of substitution between energy products	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
snrs1: Elasticity of substitution between KL and MAEN (Resource sectors)			0.2								
snrs2: Elasticity of substitution between int. goods in the resource sector			0.25								
snrs3: Elasticity of substitution between K and skilled and unskilled L			0.2								
sigmax: Elasticity between imported and domestically produced goods	2.91	3.05	5.2	2.1	10	2.8	2.95	3.97	3.30	2.95	1.90
sigmai: Armington elasticity between countries	5.81	6.10	10.4	4.2	20	5.6	5.90	7.95	6.60	5.90	3.80

	Electric Goods	Transport equipment	Other Equipment Goods	Consumer Goods	Construction	Transport (Air)	Transport (Land)	Transport (Water)	Market Services	Non Market Services
	12	13	14	15	16	17	18	19	20	21
sn1:Elasticity of substitution between KLE and MA	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
sn2: Elasticity of substitution between KL and ENG(non energy sectrors)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
sn3:Elasticity of substitution between intermediate goods	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
sn4:Elasticity of substitution between K and skilled and unskilled L	1.26	1.26	1.26	1.17	1.40	1.68	1.68	1.68	1.32	1.26
sn5:Elasticity of substitution between Energy and Electricity	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
sn6:Elasticity of substitution between energy products	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
sigmax: Elasticity between imported and domestically produced goods	4.40	3.55	3.9	3.21	1.9	1.9	1.9	1.9	2.03	1.9
sigmai: Armington elasticity between countries	8.80	7.10	7.8	6.43	3.8	3.8	3.8	3.8	4.06	3.8

Table 8 ctd.: GEM-E3 elasticities

	Coal fired	**Oil fired**	**Gas fired**	**Nuclear**	**Biomass**	**Hydro electric**	**Wind**	**PV**	**CCS coal**	**CCS Gas**	
	22	23	24	25	26	27	28	29	30	31	
sn1:Elasticity of substitution between KLE and MA	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
sn2: Elasticity of substitution between KL and ENG(non energy sectrors)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
sn3:Elasticity of substitution between intermediate goods	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
sn5:Elasticity of substitution between Energy and Electricity	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
sn6:Elasticity of substitution between energy products	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	

Income elasticity per consumption category

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
AUT	0.78	0.97	1.07	0.85	1.05	1.92	1.60	0.81	0.93	1.20	1.13	2.12	1.62	0.93
BEL	0.51	0.96	1.06	0.85	1.05	1.92	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
BGR	0.66	0.97	1.07	1.27	1.05	1.18	1.31	0.81	0.93	1.15	0.92	1.40	1.31	0.92
CYP	0.49	0.96	1.06	0.82	1.05	1.18	1.23	0.81	0.93	1.13	0.87	1.28	1.24	0.91
CZE	0.58	0.97	1.06	0.98	1.05	1.92	1.26	0.81	0.93	1.14	0.88	1.33	1.26	0.92
DEU	0.48	0.96	1.06	1.21	1.05	0.10	1.23	0.81	0.93	1.13	0.86	1.28	1.23	0.91
DNK	0.52	0.96	1.06	0.42	1.05	1.12	1.24	0.81	0.93	1.14	0.87	1.29	1.24	0.91
ESP	0.50	0.96	1.06	1.00	1.05	0.30	1.24	0.81	1.30	1.13	0.87	1.29	1.24	0.91
EST	0.63	0.97	1.06	1.11	1.05	1.18	1.28	0.81	0.93	1.15	0.90	1.36	1.28	0.92
FIN	0.53	0.96	1.06	1.09	1.05	0.99	1.24	0.81	0.93	1.14	0.87	1.30	1.25	0.92
FRA	0.49	0.96	1.06	0.87	1.05	1.64	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
GBR	0.46	0.96	1.06	1.41	1.05	0.35	1.23	0.81	0.93	1.13	0.86	1.28	1.23	0.91
GRC	0.63	0.97	1.07	1.30	1.05	0.98	1.28	0.81	1.18	1.15	0.90	1.36	1.29	0.92
HUN	0.61	0.97	1.06	1.16	1.05	0.98	1.27	0.81	0.93	1.15	0.89	1.35	1.28	0.92
IRL	0.53	0.96	1.06	0.72	1.05	0.53	1.24	0.81	0.93	1.14	0.87	1.30	1.25	0.92
ITA	0.51	0.96	1.06	1.12	1.05	0.90	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
LTU	0.63	0.97	1.07	1.33	1.05	0.90	1.28	0.81	0.93	1.15	0.90	1.36	1.28	0.92
LUX	0.39	0.96	1.06	0.67	1.05	0.65	1.22	0.81	0.93	1.13	0.85	1.26	1.22	0.91
LVA	0.65	0.97	1.07	1.27	1.05	0.90	1.30	0.81	0.93	1.15	0.91	1.38	1.30	0.92
MLT	0.55	0.96	1.06	1.24	1.05	0.53	1.25	0.81	0.93	1.14	0.88	1.31	1.25	0.92
NLD	0.48	0.96	1.06	0.60	1.05	0.56	1.23	0.81	0.93	1.13	0.86	1.28	1.23	0.91
POL	0.63	0.97	1.07	1.33	1.05	1.60	1.28	0.81	0.93	1.15	0.90	1.36	1.29	0.92
PRT	0.56	0.97	1.06	1.52	1.05	1.60	1.25	0.81	0.93	1.14	0.88	1.31	1.26	0.92
SVK	0.61	0.97	1.06	1.19	1.05	0.90	1.27	0.81	0.93	1.14	0.89	1.35	1.28	0.92
SVN	0.57	0.97	1.06	0.85	1.05	0.90	1.26	0.81	0.93	1.14	0.88	1.32	1.26	0.92
SWE	0.51	0.96	1.06	1.03	1.05	0.90	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
ROU	0.68	0.97	1.07	1.27	1.05	0.90	1.32	0.81	0.93	1.16	0.93	1.43	1.33	0.92
USA	0.35	0.96	1.06	0.63	1.05	0.66	1.21	0.81	0.94	1.13	0.85	1.25	1.21	0.91
JPN	0.49	0.96	1.06	0.86	1.05	0.66	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
CAN	0.47	0.96	1.06	0.72	1.05	0.81	1.23	0.81	1.20	1.13	0.86	1.28	1.23	0.91
BRA	0.70	0.97	1.07	0.84	1.05	1.60	1.35	0.81	1.18	1.16	0.95	1.47	1.35	0.92
CHN	0.78	0.97	1.07	0.97	1.05	0.90	1.56	0.81	0.97	1.20	1.10	1.98	1.57	0.93
IND	0.78	0.97	1.07	1.40	1.05	0.63	1.61	0.81	0.93	1.20	1.14	2.13	1.62	0.93
CRO	0.63	0.97	1.06	1.19	1.05	0.90	1.28	0.81	0.93	1.15	0.90	1.36	1.28	0.92
AUZ	0.49	0.96	1.06	0.55	1.05	0.41	1.24	0.81	0.93	1.13	0.87	1.29	1.24	0.91
FSU	0.67	0.97	1.07	0.23	1.05	1.18	1.31	0.81	0.93	1.16	0.92	1.41	1.32	0.92
ANI	0.46	0.96	1.06	0.87	1.05	1.18	1.23	0.81	0.93	1.13	0.86	1.28	1.23	0.91
ROW	0.73	0.97	1.07	1.27	1.05	0.63	1.40	0.81	0.93	1.53	1.15	1.65	1.64	1.28

Labour Supply Elasticities

	Skilled labour supply elasticity	Unskilled labour supply elasticity		Skilled labour supply elasticity	Unskilled labour supply elasticity
AUT	-1.9033	-1.9033	**MLT**	-2.7623	-2.7623
BEL	-2.8816	-2.8816	**NLD**	-0.9381	-0.9381
BGR	-3.0617	-3.0617	**POL**	-3.5428	-3.5428
CYP	-1.9279	-1.9279	**PRT**	-3.3351	-3.3351
CZE	-2.5437	-2.5437	**SVK**	-3.7013	-3.7013
DEU	-2.8856	-2.8856	**SVN**	-2.2218	-2.2218
DNK	-1.5204	-1.5204	**SWE**	-2.6716	-2.6716
ESP	-3.3289	-3.3289	**ROU**	-2.9254	-2.9254
EST	-2.2607	-2.2607	**USA**	-2.4061	-2.4061
FIN	-2.9284	-2.9284	**JPN**	-2.1140	-2.1140
FRA	-2.9600	-2.9600	**CAN**	-3.0566	-3.0566
GBR	-2.4314	-2.4314	**BRA**	-3.4556	-3.4556
GRC	-3.2623	-3.2623	**CHN**	-1.8978	-1.8979
HUN	-3.0077	-3.0077	**IND**	-3.4620	-3.4620
IRL	-2.2061	-2.2061	**CRO**	-3.5842	-3.5842
ITA	-2.7103	-2.7103	**AUZ**	-2.2249	-2.2250
LTU	-2.0108	-2.0108	**FSU**	-3.2656	-3.2656
LUX	-1.4214	-1.4214	**ANI**	-3.4987	-3.5013
LVA	-2.8216	-2.8216	**ROW**	-4.1506	-4.1516

Annex VI The Labour Market and Equilibrium Unemployment

In the standard version of the GEM-E3 model labour market is perfect in the sense that wages adjust until there is no excess labour supply and hence unemployment. The model considers the notion of voluntary unemployment through the choice of household for leisure(when the alternative version of the labour market is used). In the standard version the representation of involuntary unemployment is based on the efficiency wages approach by Shapiro and Stiglitz (1984). In the remainder of this section the main labour market imperfections leading to involuntary unemployment are presented followed by the mathematical description of the labour market extension incorporated into the GEM-E3 CGE model.

Skilled and unskilled labour

The model distinguishes labour between skilled and unskilled labour. Capital and skilled labour substitute each other (except for power generation technologies, where capital and skilled labour complement each other), at the 4^{th} level of production, while capital and skilled labour bundle are substitutes with unskilled labour at the 3^{rd} level of the production. Equilibrium unemployment is modelled for both skilled and unskilled labour and the adequate procedure is described in the section below.

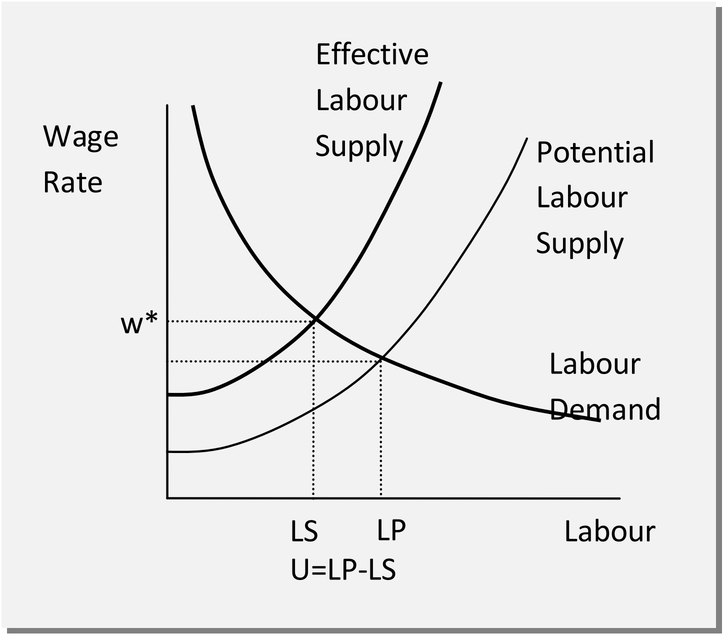
Illustration of equilibrium unemployment

The formulation of the labour market adopted in the GEM-E3 assumes the presence of imperfections and rigidities which shift the exogenous labour supply (in the alternative version the utility-derived labour supply), to the left and upwards. Wages drive the balancing of the shifted labour supply with labour demand. Thus involuntary unemployment arises as a result of the distorted labour market equilibrium.

It is assumed that, due to labour market imperfections and frictions, the employees enjoy a wage premium (a wage rent) on top of the wage rate that would correspond to equilibrium between potential labour supply and labour demand.

The wage rate premium leads to a displacement to the left of the potential labour supply curve. The displaced supply curve corresponds to effective labour supply.

Figure 13: Illustration of equilibrium unemployment



The wage rate premium is endogenous in the model and is assumed to be the consequence of the existence of Principal-Agent relations: the firms are obliged to pay a wage premium to induce employees not to shirk; as a result effective labour supply is determined through efficiency wages.

The balancing of labour demand with effective, rather than potential, labour supply implies that equilibrium unemployment is determined as the difference between potential and effective labour.

This is illustrated in Figure 13 which shows unemployment *U* as difference between potential equilibrium labour *LP* and effective labour equilibrium *LS*, corresponding to wage rate w* which includes the wage rent reflecting market imperfections.

Efficiency wages

An approach for simulating involuntary unemployment relates to the assumption that there is a negative correlation between wages and unemployment. This approach is consistent with the efficiency wages theory of Shapiro & Stiglitz (1984) which states that productivity/quality of labour has a positive correlation with wages. In periods with high unemployment firms are not motivated to offer high wages to attract higher quality labour or to increase productivity of existing workers. On the other hand, at low unemployment rates it is efficient for firms to offer wages above their equilibrium level, because they seek for increases in labour productivity and for reducing the probability of someone quitting the job and hence reducing costs from the recruitment of new personnel; see Phelps (1994), Campbell and Orszag (1998).

In the GEM-E3 model the efficiency wage approach was finally selected to be the default option for representing involuntary (equilibrium) unemployment. This modelling approach was preferred because of its empirical validation, by using for example Blanchflower and Oswald (1994), its simplicity, and the fact that it is parsimonious in parameters. The specification of efficiency wages in GEM-E3 is shown below and it is based on Shapiro & Stiglitz and Annabi (2003) approaches. The procedure is identical both for skilled and unskilled labour.

The utility function of a "shirker" worker U_S, either skilled or unskilled, is defined as:

$$r \cdot U_s = w - (q + b) \cdot (U_s - U_u)$$

where *q* is the probability of getting caught shirking, *b* the exogenous probability to quit from job, *r* the social time preference rate, *w* the wage and U_u the utility function of the unemployed. The utility function of a "non-shirker" is:

$$r \cdot U_n = w - e - b \cdot (U_n - U_u)$$

where $e \geq 0$ is the disutility from working (for the "shirker" is $e = 0$). The utility function of the unemployed is:

$$r \cdot U_u = \overline{wr} + a \cdot (U_n - U_u)$$

Where $\overline{wr}$ is the unemployment benefit and *a* the probability to get a job.

A worker decides not to be productive when $U_n \geq U_s$. This is the efficiency condition. Replacing the utility functions of the shirker and non-shirker the efficiency condition can be rewritten as:

$$w \geq \overline{wr} + e + \frac{e \cdot (a + b + r)}{q}$$

Thus efficiency wage is an increasing function of quit rate, the probability of finding a job, the interest rate and the unemployment benefit. In equilibrium the number of workers that are unemployed should equal the number of workers that fill a vacancy

$$b \cdot L = a \cdot (LS - L)$$

The unemployment rate is defined as

$$u = \frac{LS - L}{LS}$$

Thus the efficiency condition (unemployment wage functions) becomes:

$$w = \bar{w}r + e + \frac{e}{q} \cdot \left(\frac{b}{u} + r\right)$$

The efficiency condition is the labour supply function in the modified version of GEM-E3. The condition was adjusted by using the consumer price index, $PCI_{er,t}$, so as to incorporate real wages.

$$WRMEAN\_skld_{ER,T} = \frac{PCI_{ER,T}}{pcibase_{ER,T}} \cdot \left[unben\_skld_{er,t} + effort\_skld_{er,t} + \frac{effort\_skld_{er,t}}{pcaught\_skld_{er,t}} \cdot \left(stp_{er,t} + \frac{pquit\_skld_{er,t}}{UNRT\_skld_{er,t} - edelta\_skld_{er,t}}\right)\right] \quad [150]$$

$$WRMEAN\_unskld_{ER,T} = \frac{PCI_{ER,T}}{pcibase_{ER,T}} \cdot \left[unben\_unskld_{er,t} + effort\_unskld_{er,t} + \frac{effort\_unskld_{er,t}}{pcaught\_unskld_{er,t}} \cdot \left(stp_{er,t} + \frac{pquit\_unskld_{er,t}}{UNRT\_unskld_{er,t} - edelta\_unskld_{er,t}}\right)\right] \quad [151]$$

where:

$WRMEAN\_skld_{er,t}$, $WRMEAN\_unskld_{er,t}$: the wage rate of skilled and unskilled labour respectively,

$unben\_skld_{er,t}$, $unben\_unskld_{er,t}$: unemployment benefit of skilled labour and unskilled labour,

$effort\_skld_{er,t}$, $effort\_unskld_{er,t}$: disutility of effort of skilled and unskilled labour as proportion to the wage rate,

$UNRT\_skld_{er,t}$, $UNRT\_unskld_{er,t}$: unemployment rate of skilled and unskilled labour,

$pquit\_skld_{er,t}$, $pquit\_unskld_{er,t}$: exogenous probability to quit of skilled and unskilled labour, calibrated to base year data ,

$edelta\_skld_{er,t}$, $edelta\_unskld_{er,t}$: natural rate of unemployment ,

$pcaught\_skld_{er,t}$, $pcaught\_unskld_{er,t}$: probability of getting caught shirking for skilled and unskilled labour.

The implementation of involuntary unemployment in the GEM-E3 model requires additional data (i.e. unemployment levels, minimum wages etc.) that are extracted mainly from the CESifoDICE and EUROSTAT databases.

Equations [152] and [153] serve to compute the unemployment rate while the equilibrium conditions [150] and [151] in the labour market serve to compute the wage rate, which is the average nominal wage rate used to derive the labour cost of skilled and unskilled labour $PL_{\substack{pr \\ skld}}$ and $PL_{\substack{pr \\ unskld}}$ [20].

$$UNRT_{\substack{er,t \\ skld}} = 1 - \frac{\sum_{pr} LAV_{\substack{pr,er,t \\ skld}}}{POPV_{\substack{er,t \\ skld}}} \quad [152]$$

$$UNRT_{\substack{er,t \\ unskld}} = 1 - \frac{\sum_{pr} LAV_{\substack{pr,er,t \\ unskld}}}{POPV_{\substack{er,t \\ unskld}}} \quad [153]$$

where:

$$POPV_{\substack{er,t \\ skld}} = TotLabFrc_{\substack{er,t \\ skld}} \quad [154]$$

$$POPV_{\substack{er,t \\ unskld}} = TotLabFrc_{\substack{er,t \\ unskld}} \quad [155]$$

$POP_{\substack{er,t \\ skld}}$: the skilled population of each region,
$POP_{\substack{er,t \\ unskld}}$: the unskilled population of each region,

$TotLabFrc\_skld_{er,t}$, $TotLabFrc\_unskld_{er,t}$ is the total labour force (of skilled and unskilled labour) respectively, measured in million hours, drawn from WIOD database. The unit cost of skilled and unskilled labour is computed according to the average wage rate derived from the equilibrium of the labour market.

$$PL_{\substack{pr,er,t \\ skld}} = \frac{tl_{\substack{pr,er,t \\ skld}} \cdot WRMEAN_{\substack{er,t \\ skld}} \cdot XLNUM_{\substack{er,t \\ skld}}}{\left(1 - \left(txfss_{pr,er,t} - idea_{er,t}\right)\right)} \quad [156]$$

$$PL_{\substack{pr,er,t \\ unskld}} = \frac{tl_{\substack{pr,er,t \\ unskld}} \cdot WRMEAN_{\substack{er,t \\ unskld}} \cdot XLNUM_{\substack{er,t \\ unskld}}}{\left(1 - \left(txfss_{pr,er,t} - idea_{er,t}\right)\right)}. \quad [157]$$

In the alternative version of GEM-E3, when leisure is included in the model, the average wage rate and the equilibrium unemployment are computed from the following equations:

$$WRMEAN_{er,t} = \frac{PCI_{er,t}}{pcibase_{er,t}} \cdot \left[unben_{er,t} + effort_{er,t} + \frac{effort_{er,t}}{pcaught_{er,t}} \cdot \left(stp_{er,t} + \frac{pquit_{er,t}}{UNRT_{er,t}}\right)\right]$$
$$UNRT_{er,t} = 1 - \frac{\sum_{pr} LAV_{pr,er,t}}{POPV_{er,t} - LJV_{er,t}}$$
$$POPV_{er,t} = POP_{er,t} \cdot tottime_{er,t}$$

where:

$POP_{er,t}$: the population of each region,

$tottime_{er,t}$: the total available time for leisure or labour.

[20] Other model variants include a Philips curve, fixed labour supply and fixed wages.

Then the unit cost of labour and the unit cost of leisure can be computed as:

$$PL_{pr,er,t} = \frac{tl_{pr,er,t} \cdot WRMEAN_{er,t} \cdot XLNUM_{er,t}}{\left(1 - \left(txfss_{pr,er,t} - idea_{er,t}\right)\right)}$$

$$PLJ_{pr,er,t} = \left(1 - txhss_{er,t}\right) \cdot \left(1 - txdirtaxh_{er,t}\right) \cdot WRMEAN_{er,t}$$

$txhss_{er,t}$: the personal social contribution rate.

Annex VII The Environment and Emissions Module

The objective of the environment module is to represent the effects of alternative environmental policies on the global economy, namely on sectoral activity, employment, welfare etc. The aim of the introduction of an environment module is to enable the analysis in the following directions:

- Integrated analysis and impact assessment of environmental and energy policies at a European or global scale
- Representation of a larger set of environmental policy instruments at different levels: standards, taxes, tradable permits (international, national and sectoral)
- Detailed assessment of alternative climate change mitigation policies, enabled by a thorough representation of emission trading markets

The module concentrates on four major environmental problems:

(i) global warming

(ii) problems related to the deposition of acidifying emissions Integrated analysis of different environmental problems: simultaneous analysis of global warming and acid rain policy

(iii) Comparison between a source or a receptor oriented approach: damage valuation versus uniform emission reductions

(iv) ambient air quality linked to acidifying emissions and troposheric ozone concentration

Hence, energy related emissions of CO_2, NOx, SO_2, VOC and particulates, which are the main source of air pollution, are considered. NOx is almost exclusively generated by combustion process, whereas VOC's are only partly generated by energy using activities (refineries, combustion of motor fuels[21]). For the problem of global warming, CO_2 is responsible for 60% of the radiative forcing (IPCC, 1990). The GEM-E3 environment module addresses all GHGs (CO_2, CH_4, CFC, and N_2O) so as to provide a better analysis of climate change policies.

The environment module contains two sub-modules:

- a “behavioural” module, which represents the effects of different policy instruments on the behaviour of the economic agents (e.g. additive “end-of-pipe” and integrated “substitution” abatement).
- a “state of the environment” module, which uses all emission information and translates it into deposition, air-concentration and damage data. This sub-module was constructed making use of existing information or using results of other EC-projects like the ExternE. Depending on the version of the model, there is a feedback to the behaviour modules.

There are three mechanisms of emission reduction in the GEM-E3 model:

1. End-of-pipe abatement (where appropriate technologies are available)
2. Substitution between fuels and/or between energy and non-energy inputs

[21] Other important sources of VOC's are the use of solvents in the metal industry and in different chemical products but are not considered here.

3. Emission reduction due to a decrease of production and/or consumption

The dual formulation of the GEM-E3 model eases the incorporation of changes in economic behaviour due to emission or energy based environmental policy instruments. The costs of environmental policy requirements are added to the input (and consumption) prices. Intermediate demand is derived from the unit cost function which takes these extra costs into account. Similarly the demand of households for consumption categories is derived from the expenditure function, which is the dual of the utility function. Hence, the additional policy constraint is easily reflected in prices and volumes.

The model takes into account the trans-boundary effects of emissions through transport coefficients, relating the emissions in one country to the deposition/ concentration in the other countries. For secondary pollutant as tropospheric ozone, it implies considering the relation between the emissions of primary pollutants (NOx emissions and VOC emissions for ozone) and the level of concentration of the secondary pollutants (ozone).

Damage estimates are computed for each country and for the EU-15 as a whole, making the distinction between global warming, health damages and others. The figures for damage per unit of emission, deposition or concentration and per person and their valuation are based on the ExternE project results.

Mechanisms of emission reduction

There are three mechanisms which enable the reduction of emissions in the model:

1. End-of-pipe abatement: end-of-pipe abatement technologies are formulated explicitly by bottom-up derived abatement cost functions that differ between sectors, durable goods, pollutants and between countries. The marginal costs of abatement are increasing functions of the degree of abatement. These costs differ between sectors and countries according to the country- or sector-specific abatement efforts already done. End-of-pipe abatement technologies refer only to non-CO_2 emissions.
2. Substitution of fuels: as the production of the sectors is specified in nested CES-functions, there is (at least for a substitution elasticity greater than 0) some flexibility on the decision of intermediates. The input demand is linked to the relative prices of these inputs. Hence, if there is an extra cost on energy inputs, there will be a shift in the intermediate demand away from 'expensive' energy inputs towards less costly inputs. A politically imposed cost on emissions therefore drives substitution towards less emission intensive inputs, e.g. from coal to gas or from energy to materials, labour or capital.
3. Decrease of production: in a general system that covers the interdependency of agent's decision, imposing an environmental constraint (through standards, taxes or other instruments) causes additional costs to production (which is linked to the costs of substitution or abatement installation). An increasing selling price decreases demand of these goods even if this demand is inelastic to price changes (which are usually not the case) due to budget constraints. This lowers production and accordingly the demand for intermediates. Hence, there is an emission reduction due to a demand driven decline in production.

Endogenous agent's abatement decision

The firm's behaviour

The abatement activities are modelled so as to increase the user cost of the polluting input (here the price of energy) in the decision process of the firm. When an environmental tax is imposed it is paid to the government by the branch causing the pollution. This has the following implications for the energy price modelling:

- The price of energy, inclusive abatement cost and taxes, is used in the decision by the firm on production factors (at the energy level and implicitly at the level of aggregates, according to the CES levels of aggregation); it represents the user's cost of energy
- The price of energy, exclusive taxes and abatement cost[22], is used to value the delivery of the energy sectors to the other sectors
- A price for the abatement cost per unit of energy has been defined, because the abatement cost is defined in constant price

In the modelling of the abatement activities, installing abatement technologies has been considered as an intermediate input for the firms ($abiov_{i,ii}$) and not as investment demand of the firms. The total delivery for abatement is added to the intermediate demand and these inputs are priced as other intermediate deliveries. The major advantage of this formulation is that with this framework the abatement costs do not increase directly GDP as it would if modelled as investment but only indirectly as additional intermediate demand[23]. For the purpose of introducing an investment, a depreciation and replacement mechanism would have to be introduced. The user's cost of the abatement equipment would have to be added to the capital income, avoiding however any double counting.

Consumer's behaviour

Consumer's behaviour modelling is similar to the one used for the firm (for consumers it refers only to energy related emissions). The difference lies on the payment of the environmental taxes to the government. While in the case of firms, the environmental taxes are paid by the branch causing the pollution, for households the tax is paid by the branch delivering the product causing pollution to the household.

The environmental tax is therefore treated as the other indirect taxes paid by households. This has the following implications for the modelling of the price equations:

- The price of energy in the consumer allocation decision, includes the abatement cost and the tax (it is modelled as a user's cost of energy)
- The price of delivery of energy to the household includes the pollution and/or energy tax
- A price for the abatement cost is defined in the same way as for the branches

[22] i.e. it is the same price as the one in the model without environmental module.

[23] This approach may be subject to limitations as the abatement costs have to be paid in every period leading to possible overestimation of abatemen cost/ permit prices.

Modelling end-of-pipe abatement costs

The average abatement cost reflects annualized costs and the value for the parameters in the equation are based on the estimated technical data. Table 9 describes the different sets used in the following equations and in GEM-E3:

Table 9: Definition of the GEM-E3 sets relevant to the environment module

Set name in equations	Set name in GEM-E3	Definition
r	Cott	countries
	Stime	time
i,ii	pr,br	branches
j,jj	lnd,dg	Durable (dg) and non-durable (lnd) consumption categories
po1	po1	CO_2, NO_X, SO_2, VOC, PM, CH_4, N_2O, PFC, HFC, SF6, $CO_{2\text{-Cement}}$
aghg	Aghg, subset of po1	Greenhouse gases: CO_2,CH_4,N_2O,PFC,HFC,SF_6,$CO_{2\text{-Cement}}$
poab	poab, subset of po1	Abated through end-of-pipe technologies : NO_X, SO_2, VOC, PM, CH_4, N_2O, PFC, HFC, SF6, $CO_{2\text{-C}}$
poabe	poabe, subset of poab	Related to fossil fuel combustion: SO_2, NOx, VOC, PM
pre	pre, subset of pr	all energy branches with emissions
cc	Cc	Club participating in emission trading scheme

Abatement decision

In order for the firm and/or the household to decide on the optimal level of abatement through end-of-pipe technologies, the endogenous or exogenous price of emission allowances (opportunity cost for the firm and/or household to emit less) is taken into consideration. The decision is taken so as to abate emissions, according to the marginal abatement cost curve, up to a level which is seen cost-effective, i.e. up the level that the cost to abate the last tone of emissions equals the price of emission allowances.

Above that level, the firm and/or household find it most cost-efficient to emit than abate. End-of-pipe technologies can only abate non-CO_2 emissions since carbon dioxide emissions are directly related to fuel combustion and can only be abated through fuel substitution (or power generation via non-emitting technologies like renewable power) or through a reduction in production (or improved energy efficiency).

The firms decision on whether to abate or to pay taxes can be derived from its profit maximisation, as described in its generalised format below.

$\max \prod_j,$

where:

$\prod_j = PX_j \cdot X_j - VC_j$ (with VC_j as variable cost function).

The variable cost function VC_s is then given by:

$VC_{j=} \sum_{i=1}^{n+2} v_i \cdot PY_i,$

where:

$v_{i,j}$: is intermediate demand of input *i* by sector *s* assuming that *i* includes labour and capital (*n+1*and *n+2*).

To ease the notation in the following presentation an input price PY_j is defined that includes emission and/or energy-taxes as well as indirect taxes[24]. This price is associated to the GEM-E3 variables $PIO_{i,j}$, PK_j *and* PL_j:

$$PY_i = (1 + t_i) \cdot PY_i + c_j^{en} \cdot ec_i \cdot x_{i,j} + \sum_{po1} \left[ef_{poab,i,j} \cdot \mu_{i,j} \cdot \left(c_{poab,j}^{ab}(a_{poab,j}) \cdot a_{poab,j} + c_{poab,j}^{ef}(a_{poob,j}) \cdot (1 - a_{poab,j}) \right) \right]$$

Where

$poab_{,j}$: all pollutants included in the model,

[24] Assuming linear-homogenity of the cost function $VC_s\left(X_s, \vec{P}Y^{act}, a_s, \overline{K}_{fix}\right)$ with respect to output quasi-fixed capital stock) eases the solution of the maximization problem considerably. (see Schrooder, 1991)

c_j^{en}: the energy related tax, ec_j: is combustible energy component of input, corresponding to $aer_{pre,i}$ parameter of GEM-E3.

$x_{i,j}$:the energy related input *i* for production of sector *j*, corresponding to the $iov_{pre,i}$ variable of the GEM-E3,

$ef_{proab,i,j}$: the emission factor for pollutant *po1* from energy input *i* for the production of sector *j* ,

$\mu_{i,j}$: energy related coefficient of energy input *i* to sector *j* ,

$a_{proab,j}$: the level of abatement , corresponding to $aa_{i,proab}$ variable of the GEM-E3,

$c^{ab}_{poab,j}(a_{poab,j})$: the cost of abatement as a function of the level of abatement,

$c^{ef}_{poab,j}(a_{poab,j})$: the cost of emitting as a function of the level of abatement (or else as a function of the level of actual emissions).

The first order conditions of the profit maximizing firm serve to determine supply and the degree of abatement. For the description of the environmental module only the latter is of interest.

As the abatement costs are not distinguished by inputs, the formula for the optimal degree of abatement of pollutant *po1* can be reduced to the following expression:

$$\frac{\partial G_j}{\partial a_{po1,j}} = -\frac{\partial VC_j}{\partial \vec{P}Y} \cdot \frac{\partial \vec{P}Y}{\partial a_{poab,j}} = -\sum_i v_{i,j} \cdot \frac{\partial\left(\left[\left(c^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ef}_{poab,j}(a_{poab,j}) \cdot (1-a_{poab,j})\right)\right] \cdot \sum_i ef_{poab,i,j} \cdot \mu_{i,j}\right)}{\partial a_{poab,j}} = -\sum_i \big(v_{i,j} \cdot ef_{poab}, i, j \cdot \mu_{i,j}\big) \cdot \frac{\partial\left(c^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ef}_{poab,j}(a_{poab,j}) \cdot (1-a_{poab,j})\right)}{\partial a_{poab,j}} = 0 \Rightarrow \frac{\partial\left(c^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ef}_{poab,j}(a_{poab,j}) \cdot (1-a_{poab,j})\right)}{\partial a_{poab,j}} \Rightarrow$$

$$mc^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ab}_{poab,j}(a_{poab,j}) + mc^{ef}_{poab,j}(a_{poab,j}) \cdot (1 - a_{poab,j}) - c^{ef}_{poab,j}(a_{poab,j}) \overset{!}{=} 0$$

Hence, in case of an exogenous emission tax rate of $t^{env}_{poab,j}$ ($c^{ef}_{poab,j} = t^{env}_{poab,j}$ and $mc^{ef}_{poab,j} = 0$) the (cost minimising) degree of abatement $a_{poab,j}$ can be derived (numerically) by the following implicit equation, which is found in the model (as described below).

$$\frac{\partial G_j}{\partial a_{poab,j}} = mc^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ab}_{poab,j}(a_{poab,j}) - t^{env}_{poab,j} = 0$$

The abatement decision of households can be derived in a similar way. To reduce the complexity of the analytical solution, it is assumed that only the fixed part of the linked non-durable demand is affected by the end-of-pipe emission reduction measures. Hence, the degree of abatement is independent of the prices and quantities of the linked consumption.

The derivation of the cost minimising degree of abatement for household emissions can be reduced according to the following expressions[25]:

[25] This assumption is not very restrictive as the disposable part of the linked non-durables is typically very small (around 5 to 10 %).

$$\frac{\partial\tilde{u}}{\partial a_{poab,j}} = \frac{\partial\tilde{u}}{\partial z_j}\frac{\partial z_j}{\partial p_{dur_j}}\frac{\partial p_{dur_j}}{\partial a_{poab,j}} = 0 \Rightarrow \frac{\partial p_{dur_j}}{\partial a_{poab,j}}$$
$$= \sum_l \theta_{l,j} \cdot ef_{poab,l,j} \cdot \mu_{l,j} \cdot \frac{\partial\left(c^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ef}_{poab,j}(a_{poab,j}) \cdot (1 - a_{poab,j})\right)}{\partial a_{poab,j}} = 0$$
$$\Rightarrow mc^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ab}_{poab,j}(a_{poab,j}) + mc^{ef}_{poab,j}(a_{poab,j}) \cdot (1 - a_{poab,j}) - c^{ef}_{poab,j}(a_{poab,j}) \overset{!}{=} 0$$

Under an exogenous emission tax $t^{env}_{poab,j}$ ($c^{ef}_{poab,j} = t^{env}_{poab,j}$and $mc^{ef}_{poab,j} = 0$), the optimal degree of abatement $a_{poab,j}$ is given by the following implicit equation:

$$mc^{ab}_{poab,j}(a_{poab,j}) \cdot a_{poab,j} + c^{ab}_{poab,j}(a_{poab,j}) = t^{env}_{poab,j}$$

The marginal abatement cost function used in GEM-E3 for emissions of SO_2, NOx, VOC, PM, i.e.

$mc^{ab}_{poabe,j}(a_{poabe,j})$, is the following:
$$mc^{ab}_{j,poabe} = cabf1_{j,poabe} * (1 - aa_{j,poabe})^{cabf2_{j,poabe}}$$

where:

$cabf1_{i,poabe}$, $cabf2_{i,poabe}$, $cabf3_{poabe}$: are the parameters of the marginal abatement cost function estimated through bottom-up engineering.

While for emissions related to industrial processes, i.e. $CO_{2\text{-Cement}}$, CH_4, N_2O, PFC, HFC, SF_6 the marginal abatement cost function estimated by IIASA and EPA data is:

$$mc^{ab}_{j,poabe} = mac1 \cdot (ee^{AA} - 1)$$

The decision on the level of abatement taken by the firms, namely $aa_{i,poab}$, can be found in the GEM-E3 model as described above after solving the firm's profit maximization problem. In particular, the level of abatement depends on the type of pollutants so as to equalize the marginal cost of abatement to the price of emission allowances or tax.

In a mixed complementarity problem formulation (MCP), like the GEM-E3, a set of equations can be written as inequalities. The inequality ensures the zero profit condition according to the Kuhn-Tucker conditions. In particular, in order to determine the optimum level of abatement for a firm that takes end-of-pipe measures and ensure the zero profit condition, the cost of abatement should be greater to or equal to the revenues from abating emissions.

The complementary slackness requires that the level of abatement is above zero if the inequality holds as an equality, otherwise the necessary condition for optimality requires that the choice variable (here $aa_{i,poab}$) is zero since the cost of abatement is greater than the revenues (or opportunity cost) from reducing the emissions.

The equations below are categorized according to the pollutant to be abated, namely poabe, poabx emissions are treated differently.

$$MCGHG_{poab,br,er,t} \leq TXENV_{poab,br,er,t\ if\ poabx=poab} \quad [158]$$

where:

$MCGHG_{poab,br,er,t}$: the marginal abatement cost of non-energy related greenhouse gas emissions

In order to calculate the cost of emission abatement through the use of end-of-pipe technologies, equations [159] and are used. The real cost per unit of abatement, namely $CABAVV_{poab,br,er,t}$ is then included in the cost of production.

$$CABAVV_{poab,br,er,t} = mac1_{er,poab,br,t} \cdot \frac{\left(e^{AA_{poab,br,er,t}} - AA_{poab,br,er,t} - 1\right)}{tabcost_{poab,pr,er,t} \cdot PIO_{pr,er,t}} if\ poabx = poab \qquad [159]$$

Demand for intermediate inputs to meet abatement purposes, $ABIOV_{pr,br,er,t}$ in the case of firms, is added directly to domestic demand for goods $Y_{pr,er,t}$. The following equation estimates the respective volume of demand:

$$ABIOV_{pr,br,er,t} = \sum_{poabx} \left(tabcost_{poabe,pr,er,t} \cdot mec_{poabx,br,er,t} \cdot XD_{br,er,t} \cdot CABAVV_{poabx,br,er,t} \cdot AA_{poabx,br,er,t}\right) \qquad [160]$$

where:

$tabcost_{poabe,pr,er,t}$: is the share of energy component (combustible) of intermediate input (in PJ/monetary unit)

$mec_{poabx,br,er,t}$: the emission coefficient per unit of production (in $MtnCO_2$/monetary unit),

Firms endogenously decide for the optimal level of emission abatement through end-of-pipe technologies, fuel substitution and/or the decline of production and/or consumption. The remaining emissions of each sector, $EMMBR_{po1,br,er,t}$, , post abatement ($AA_{poabx,br,er,t}$) and the emissions of households ($EMMHLND_{po1,lnd,br,er,t}$) are calculated below:

Firms

$$EMMBR_{po1,br,er,t} = \begin{cases} \sum_{pre} bec_{po1,pre,br,er,t} \cdot aer_{pre,br,er,t} \cdot IOV_{pre,br,er,t} & if\ po1 = poem \\ \left(1 - AA_{po1,br,er,t}\right) \cdot mec_{po1,br,er,t} \cdot XD_{br,er,t} & if\ po1 = poabx \end{cases} \qquad [161]$$

where:

$mec_{po1,br,er,t}$: greenhouse gasses related emission factor

$XD_{br,er,t}$: domestic production

$bec_{po1,pre,br,er,t}$: emissions coefficient per monetary unit in the branch level

$aer_{pre,br,er,t}$: share of energy consumption with emissions in the branch level

Households

$$EMMHLND_{po1,br,er,t} = \{bechdg_{po1,lnd,dg,er,t} \cdot aerhdg_{lnd,dg,er,t} \cdot LLNDC_{lnd,dg,er,t} \quad for\ po1 = poem \qquad [162]$$

Internalization of the cost of emitting/polluting

The price of production is corrected so as to include the cost of abatement technologies for process-related emission reductions ($cabav_{poabx} * aa_{poabx} * mec_{poabx}$) as well as to include the expenditure due to permit purchase or tax payment per unit of production ($txenv * (1 - aa_{poabx}) * mec_{poabx}$). If grandfathering of allowances is considered, the value of the free permit endowment, $psale_{br,er,t}$, is subtracted from the unit cost of production (if the switch parameter $swupr_t$=0, as is explained in the section below):

$$\begin{aligned} PD_{br,er,t} = {} & PDBSR_{br,er,t} - PSALE_{br,er,t} \\ & + \sum_{poabx} \Big(TXENV_{poabx,br,er,t} \cdot (1 - AA_{poabx,br,er,t}) + AA_{poabx,br,er,t} \\ & \cdot \sum_{pr} tabcost_{poabx,pe,er,t} \cdot PIO_{pr,er,t} \cdot cabavv_{poabx,br,er,t} \Big) \cdot mec_{poabx,br,er,t} \end{aligned} \qquad [163]$$

where:

$PDBSR_{br,er,t}$: the cost of production deriving from the firm's production function,

$PSALE_{br,er,t}$: the value of free permit endowment per unit of production.

As regards the internalization of environment-related external costs in the household decision, the user cost of linked non-durables is corrected so as to include the cost of abatement technologies for energy-related emissions and the cost of permit purchase or tax payment for the non-abated emissions:

$$\begin{aligned} & PUHCFVDG_{lnd,dg,er,t} \\ & = \sum_{pr} \left(thcfv_{pr,lnd,er,t} \cdot \frac{qtch_{pr,lnd,er,t}}{e^{tgqtch}} \cdot PHC_{pr,er,t} \right) \\ & + \sum_{poab} \big(TXENVHDG_{poab,dg,er,t} \cdot (1 - AAHDG_{poab,dg,er,t}) \cdot bechdg_{poab,dg,er,t} \\ & \cdot aerhdg_{lnd,dg,er,t} \big) + \sum_{poab} \big(CABAVHDG_{poab,dg,er,t} \cdot bechdg_{poab,dg,er,t} \cdot aerhdg_{lnd,dg,er,t} \big) \\ & + \sum_{poem} \big(TXENVHDG_{poem,dg,er,t} \cdot bechdg_{poem,dg,er,t} \cdot aerhdg_{lnd,dg,er,t} \big) \end{aligned} \qquad [164]$$

where:

$thcfv_{pr,lnd,er,t}$: the share parameter of industry delivery to private consumption,

$PHC_{pr,,er,t}$: the price of delivery to private consumption,

$qtch_{pr,lnd,,er,t}$: Efficiency

$bechdg_{,poab,dg,,er,t}$: the emission coefficient per monetary unit,

$aerhdg_{lnd,dg,er,t}$: the share of energy consumption with emission per durable.

The cost of emitting is internalised in the firm's and/or household's optimization through the price of emission permits and is estimated as follows:

Firms

$$TXENV_{po1,br,er,t} = \sum_{cc} PPCLUB_{po1,br,er,t} \cdot swclubbr_{po1,br,er,cc,t}$$ if endogenously calculated [165]

$$TXENV_{po1,br,er,t} = \left(txem_{po1,br,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \right) \cdot swtxexobr_{po1,br,er,t}$$ if imposed exogenously [166]

$$TXENV_{po1,br,er,t} = \left(txemeu_{po1,br,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \right) \cdot swtxexobr_{po1,br,er,t}$$ if imposed exogenously for a cluster of countries [167]

Households

$$TXENVHDG_{po1,dg,er,t} = \sum_{cc} PPCLUB_{po1,cc,t} \cdot swclubh_{po1,dg,er,cc,t}$$ if endogenously calculated [168]

$$TXENVHDG_{po1,dg,er,t} = \left(txemhdg_{po1,dg,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \right) \cdot swtxexoh_{po1,dg,er,t}$$ if imposed exogenously [169]

$$TXENVHDG_{po1,dg,er,t} = \left(txemeuhdg_{po1,dg,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \right) \cdot swtxexoh_{po1,dg,er,t}$$ if imposed exogenously for a cluster of countries [170]

where:

$PPCLUB_{po1,br,er,t}$: the endogenous carbon tax and

txempo1,br,er,t, txemhdgpo1,br,er,t: the exogenous carbon tax

Endogenous or exogenous carbon tax

In GEM-E3 a GHG reduction policy can be implemented either through exogenous tax enforcement (thereby the level of the exogenous tax is given in advance but the level of emission reductions is unknown and is endogenously estimated), or through an exogenous implementation of an emission cap, namely an endogenous tax enforcement (thereby the level of the tax is originally unknown and endogenously estimated in order to achieve a specific emission reduction target). The estimation of the endogenous tax level ensues as the clearing price of demand and supply for emission permits.

The respective equations for supply of permits are given by equation [171].The available permits for the club are calculated and allocated according to the reduction target relative to emissions in 2005, set on a country or on a regional level ($dproeu_{po1,cc}$). The components of each equation are multiplied by a “switch” parameter so as to ensure zero permit supply in case of zero value for the “switch” parameter, as mentioned above.

Supply of permits:

$$SUPPERFEU_{po1,cc,t} = \sum_{er} \left[(1 - dporeu_{po1,cc,t}) \cdot \sum_{br} \left((emmbr\_2005_{po1,br,er} \cdot swclubbr_{po1,br,er,cc,t}) + \sum_{lnd,dg} (emmhlnd\_2005_{po1,lnd,dg,er} \cdot swclubh_{po1,dg,er,cc,t}) \right) \right] \quad [171]$$

where:

$dporeu_{po1,cc,t}$: the reduction target at the club level for permit allocation or cap on trade,

$emmbr\_2005_{po1,br,er}$: the emissions by branch in 2005.

$emmhlnd\_2005_{po1,br,er}$: the emissions of households in 2005.

The respective equation for the demand of permits is given below:

$$DEMPEREU_{po1,cc,t} = \sum_{er} \left(\sum_{br} \left((EMMBR_{po1,br,er,t} \cdot swclubbr_{po1,br,er,cc,t}) + \sum_{lnd,dg} (EMMHLND_{po1,lnd,dg,er,t} \cdot swclubh_{po1,dg,er,cc,t}) \right) \right) \quad [172]$$

The market clearance of the emission permit market results in $PPCLUBAG_{po1,br,er,t}$ or else in the price of emission permits. The equations below describe the market clearance. In the MCP formulation, these equations are given as inequalities, ensuring that if the inequality holds, i.e. if supply is larger than demand, then the dual price of the carbon permits equals to zero (complementary slackness).

$$\sum_{aghg} SUPPERFEU_{aghg,cc,t} \geq \sum_{aghg} DEMPEREU_{aghg,cc,t} \qquad \text{dual variable PPCLUBAG} \quad [173]$$

with:

$$PPCLUB_{po1,cc,t} = PPCLUBAG_{po1,cc,t} \cdot swclub_{po1,cc,t} + QUOTTRB_{po1,cc,t} - QUOTTRS_{po1,cc,t} \quad [174]$$

where:

$swclub_{po1,cc,t}$: switch for the introduction of endogenous tax or permit market,

$QUOTTRB_{po1,cc,t}$: the price of permits bought in the international market and

$QUOTTRs_{po1,cc,t}$: the price of permits sold in the international market

The switches $swtxexoh_{po1,dg,er,}$, $swtxexobr_{po1,br,er,t,}$, enable the exogenous imposition of a carbon price for a period of time equal to the endogenous carbon price of a selected previous year or of the reference case carbon price. Trade of permits outside the club is activated via the *swtrcc* parameter. $QUOTTRB_{po1,cc,t}$ and $QUOTTRs_{po1,cc,t}$ are the unit cost of buying and selling emission permits, respectively, outside the club.

$$trshareb_{po1,cc,t} \cdot supperfeu_{po1,cc,t} \geq -(supperfeu_{po1,cc,t} - DEMPEREU_{po1,cc,t}) \quad [175]$$

$$trshares_{po1,cc,t} = supperfeu_{po1,cc,t} - DEMPEREU_{po1,cc,t} + \sum_{eu} mapClub_{eu,cc} \cdot nallocc_{po1,eu,cc,t} + \sum_{eu} mapClub_{eu,cc} \cdot temperalcc_{po1,eu,cc,t} \quad [176]$$

where,

$trshareb_{po1,cc,t}$: the cap on buying permits

$trshares_{po1,cc,t}$: the cap on selling permits

$nalloc_{po1,eu,cc,t}$: the national allocation of permits

$temperallc_{po1,eu,cc,t}$: the reference emissions in the permit system in the baseline scenario

$mapClub_{po1,cc}$: the parameter used to assign a country to a club

GEM-E3 simulation features for environmental policy

According to the environmental policy under analysis, GEM-E3 features a selective activation of equations and respective variables that enable the appropriate simulation of policies. In this way, detailed alternative policies can be assessed as regards, for example, the allocation of emission allowances, the participation of country clusters in common emission reduction clubs, the recycling of government revenues from the sale of emission allowances and other detailed policy features. The activation of the appropriate equations is undertaken by means of specified "switch" parameters that take the value of one (1) for activation of the respective equation or the value of zero (0) for deactivation of the respective equation.

In

Table 10, the different simulation possibilities are presented together with the respective "switch" parameters. Parameters are expressed exactly as in the GAMS code, including the original names of the sets of the parameters.

Table 10: Description of "switch" parameters for GEM-E3 scenario definition

"switch" parameter	**Activated feature if "switch" parameter value equal to 1**
	Environmental Switches
SWCLUBAG(cc,stime)	Introduction of permit market or tax for aggregate pollutant, thereby endogenous permit price is equal for all mitigated pollutants
SWHAAGHG(cc,stime)	The switch value is not assigned directly by the user, but instead is estimated in the scenario formulation according to the supply and demand of emission permits and defines the value of other switch parameters (e.g. swclubag, swclubbr). Thereby, if supply of permits is larger than demand from reference case emissions (i.e. there is a "hot air" supply of allowances) the switch parameter equals to zero (0).
SWTXEXOBR(po1,br,cott,stime)	Introduction of exogenous carbon tax (*txem*) on firms, for selective pollutants (*po1*) in selective activities (*br*) and countries(*cott*) in time (*stime*)
SWTXEXOH(po1,dg,cott,stime)	Introduction of exogenous carbon tax (*TXEMHDG*) on households, for selective pollutants (*po1*) in selective durable consumption categories (*dg*) and countries (*cott*) in time (*stime*)
SWCLUBBR(po1,br,cott,cct,stime)	Introduction of an emission reduction target (*dporbr* on a branch level or *dporeu* on a regional level) on club (*cct*) relative to 2005 emissions for selective pollutants (*po1*) in selective activities (*br*) and countries (*cott*) in time (*stime*)
SWCLUBH(po1,dg,cott,cc,stime)	Introduction of an emission reduction target (*dporh*) on households, for selective pollutants(*po1*) in selective durable consumption categories (*dg*) and

	countries (*cott*) in time (*stime*)
SWONPOR(po1,pr,cott,stime)	Allocation of emission permits with grandfathering, i.e. for free according to emissions of 2005
SWUPR(stime)	Enables the use of revenues from free emission permits to be added to capital income. If zero then revenues from free permits reduce the unit cost of production of each branch
SWBSCC(po1,cc,stime)	Introduction of burden sharing mechanism in a club according reference scenario emissions and introduction of specific treatment of additional revenues due to “hot air” permit supply
SWTRCC(po1,cott,cc,stime)	Emissions trading take place only within the country of a club and not at a regional club level. “Switch” parameter also enables a trade restriction on buying and/or selling of emission permits on a country level
SHAUCTBR(po1,pr,cott,stime)	Introduction of gradual transition from free emission permits to purchase of each permit through auction type mechanisms. The parameter can take values between 0 and 1, showing the share of permits that will not be given for free
SWCLUB(po1,cct,stime)	Introduction of an endogenous carbon tax computed according to the respective reduction emissions target set by the user
SW_RES(pr,stime)	Activation of depletable resources, allowing for the computation of an international price
Budget balancing Instruments	
SWONCA(cott,stime)	Interest rate endogenously estimated so as the current account deficit/surplus as a percentage of GDP, expressed in current prices, remains unchanged in all scenarios. In that way, the country is not allowed to increase its borrowing in order to comply to the environmental policy
SWONCAEU(cott,stime)	Interest rate endogenously estimated so as the current account deficit/surplus as a percentage of GDP, expressed in current prices, for the EU zone remains unchanged in all scenarios.
SWONCAFIX(cott,stime)	Interest rate endogenously estimated so as the current account deficit/surplus as a percentage of GDP, expressed base year prices, remains unchanged in all scenarios. In that way, the country is not allowed to increase its borrowing in order to comply to the environmental policy
SWONID(cott,stime)	Constraint to keep the government’s deficit/surplus as a percentage of GDP unchanged in all scenarios. Option can be used for recycling to the economy of the extra government revenues e.g. from permit sales in case of auctioning. The dual variable (*IDEA*) serves for the reduction of social security contributions.

SWTRHOUS(cott,stime)	Constraint to keep the government's deficit/surplus as a percentage of GDP unchanged in all scenarios. The option can be used for recycling to the economy of the extra government revenues e.g. from permit sales in case of auctioning. The dual variable (*TRHOUS*) serves as a lump-sum transfer to households.

Grandfathering (free) allowances and burden sharing

One method of permit allocation is the supply of free allowances through grandfathering (allocation of permits based on base year emissions) or other type of sectoral distribution. In the GEM-E3 this simulation is enabled with the "switch" parameter $swonpor_{po1,br,er,t}$ which allows for transfer of the value of emission permits to the firms and/or households by a respective reduction of the production cost or increase of the capital income for firms and by a transfer of value from the government to households.
Equation [177] gives the total value of grandfathered emission permits for firms, i.e. the supply of allowances as in equation [171] multiplied by the price of permits $PPBR_{po1,br,er,t}$:

$$SALEP_{po1,br,er,t} = \begin{cases} EMMBR_{po1,br,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBBR_{po1,br,er,cc,t} & if\ not\ swprimalloc_{po1,br,er,t} \\ (1 - dporbr_{po1,br,er,t}) \cdot emmbr\_2005_{po1,br,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBBR_{po1,br,er,cc,t} \cdot swonpor_{po1,br,er,t} & if\ \ swpr \\ nallo\_br_{po1,br,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBBR_{po1,br,er,cc,t} \cdot swonpor_{po1,br,er,t} & if\ \ swprimalloc_{po1,br} \end{cases} \quad [177]$$

where

$dporbr_{po1,br,er,t}$: the reduction target at branch level for permit allocation or cap on trade

$nallo\_br_{po1,br,er,t}$: the allocation of permits to branches

$SALEP_{po1,br,er,t}$ is then used to calculate $PSALE_{br,er,t}$ which will be subtracted from the unit cost of production $PDBSR_{br,er,t}$ of branch *BR* or is directly added to capital income if the "switch" parameter $swupr_t$ is activated.

When $swupr_t = 0$, the user should ensure that there is no possibility of having negative unit cost of production if the unit cost of production $PDBSR_{br,er,t}$ is smaller than $PSALE_{br,er,t}$.

$$PSALE_{br,er,t} = \frac{\sum_{po1}(1 - SHAUCTBR_{po1,br,er,t}) \cdot SALEP_{po1,br,er,t}}{XD_{br,er,t}} \quad [178]$$

Equation below calculates the total value of grandfathered emission permits for households:

$$SALEPH_{po1,br,er,cc,t} = \begin{cases} \sum_{dg}\left(EMMHLND_{po1,lnd,dg} \cdot swonporh_{po1,dg,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBH_{po1,dg,er,cc,t}\right) & if\ not\ swprima \\ \sum_{dg}(1 - dporh_{po1,br,er,t}) \cdot emmhlnd\_2005_{po1,lnd,dg} \cdot swonporh_{po1,dg,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBH_{po1,dg,er,cc,t} & if\ s \\ nallo\_hh_{po1,lnd,er,t} \cdot swonporh_{po1,dg,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBH_{po1,dg,er,cc,t} & if\ swprimalloch_{po1,} \end{cases} \quad [179]$$

where:

$dporh_{po1,br,er,t}$: the reduction target for permit allocation or cap on trade for households

$nallo\_hh_{po1,lnd,er,t}$: allocation of permits to households

The remaining expenditure (revenue) each firm has to make (receive), after the allocation of free emission allowances, $SALEP_{po1,br,er,t}$ in order to comply with the emission target is given by equation below:

$$BUSAT_{po1,br,er,t} = EMMBR_{po1,br,er,t} \cdot \sum_{cc} PPCLUB_{po1,cc,t} \cdot SWCLUBBR_{po1,br,er,cc,t} - SALEP_{po1,br,er,t} \quad [180]$$

The expenditure (revenue) each household has to make (receive), after the allocation of free emission allowances , $SALEP_{po1,br,er,t}$ in order to comply with the emission target is given by equation below:

$$BUSATH_{po1,lnd,er,t} = \sum_{dg} EMMHLND_{po1,lnd,dg,er,t} \cdot swonporh_{po1,dg,er,t} \cdot TXENVHDG_{po1,dg,er,t} - SALEPH_{po1,lnd,er,t} \quad [181]$$

$BUSAT_{po1,br,er,t}$ is then received by the government as revenue, $FGRB_{gvb,br,er,t}$, due to the enforcement of an environmental tax. If $SHAUCTBR_{po1,br,er,t}$, i.e. partial auctioning of the allowances, is also activated, then $BUSAT_{poi,br,er,t} + SHAUCTBR_{poi,br,er,t} \cdot SALEP_{poi,br,er,t}$ is received by the government.

$BUSAT_{po1,br,er,t}$ also acts as a transferring mechanism of value between the world and the government. In particular, if there is no “hot air” in the permit allocation system (i.e. original permit supply is higher than the actual target), certain countries of the same emission reduction club will present a positive $BUSAT_{po1,br,er,t}$ while the rest will present a negative $BUSAT_{po1,br,er,t}$ value, depending on the original permit allocation and each country’s endogenous decision on emission abatement. Thereby, countries with positive $BUSAT_{po1,br,er,t}$ transfer this value to the governments with negative $BUSAT_{po1,br,er,t}$, not on a bilateral basis but rather through the transfers of the government to/from the world.

“Hot air” permit supply

The case of “hot air” permit supply is treated specifically in GEM-E3 model. If there is “hot air” permit supply, i.e. larger permit supply than actual baseline emissions, then half of the respective value $SALEP_{po1,br,er,t}$ is transferred from the government to the household (lump-sum transfer) and the rest is transferred from the government to the world.

In this way, the government has no additional revenues due to “hot air” permit supply. Equation [182], enables the transfer of “hot air” additional revenues from oversupply of permits ($SALEP_{,PO1,BR,ER,T}$ value) and are not activated if the emission reduction target is set on a country level (i.e. $dporbr_{po1,br,er,t} \neq 0$).

$$SALEPG_{po1,br,er,t} = \sum_{cc} \left(NALLOCC_{po1,er,cc,t} - TEMPERALCC_{po1,er,cc,t}\right) \cdot PPCLUB_{po1,cc,t} \quad [182]$$

$SALEPG_{po1,br,er,t}$ is calculated only if:

$NALLOCC_{po1,er,cc,t} > 0$ and $NALLOCC_{po1,er,cc,t} - TEMPERALCC_{po1,er,cc,t} > 0$.

Recycling options for permit revenues

In microeconomic theory, the distortionary effect of taxes in the economy can be reduced by the recycling of revenues occurring from a second tax (here the carbon permits) with growth-enhancing effects on the longer-run. Such efficiency gains could lead to the double dividend effect if addressed optimally. A simple application of this “efficiency value” of the carbon permits is the “employment dividend” according to which the distortions created by taxes on labour can be reduced.

The economic impacts of climate policies rely on the choice of the revenue recycling options. In the GEM-E3 model, two recycling options can be found: i) the lump-sum transfer to the household income and ii) the reduction of the social security contribution of employees. Both recycling mechanisms are based on the idea that government surplus/deficit as a percentage of GDP should remain the same both in the scenario and the reference case. Thereby, the dual price of the constraint which ensures the required percentage of government surplus/deficit to GDP is added according to the selected recycling option, as explained below.

Social security contribution: This recycling option is activated by the switch parameter $swonid_{er,t}$. Variable $IDEA_{er,t}$ is the dual variable of the constraint. This dual variable enters the equation of the unit labour cost by reducing[26] the social security contribution $txfss_{pr,er,t}$, and is also incorporated in the transfers received by the government from other sectors (firms), so that when the constraint is activated, the government receives reduced revenues from firms’ social contributions. This recycling option can potentially have positive effects on employment, since the firms see a reduced cost of labour.

$$SURPL_{se,er,t} = SURPLGRFFX_{er,t} \cdot VU_{er,t} \quad for\ SE = G \quad [183]$$

Lump-sum transfer to household: This recycling option is activated by the switch parameter $swtrhous_{er,t}$. The variable $TRHOUS_{er,t}$ is the dual variable of the constraint and enters the

[26] It should be taken under consideration that the constraint could ensue in an increase of social security contributions and a subsequent increase in unit labour costs in case of a reduction in government revenues, in spite of the carbon permit revenues, which would lead in a negative dual variable $idea_{er,t}$. In case of a larger $idea_{er,t}$ than $txfss_{pr,er,t}$ the unit cost of labour is still reduced.

transfers of the government to the households, thereby increasing the disposable income of the households.

$$SURPL_{se,er,t} = SURPLGRFFX_{er,t} \cdot VU_{er,t} \quad for\ SE = G \qquad [184]$$

Annex VIII Bottom – up representation of the electricity sector

CGE models have been criticised for their simplified modelling approach of the energy system. The usual CGE representation of the energy production by means of aggregate production functions fails to capture crucial characteristics of the sector reducing the credibility of simulations related to energy policies and technology dynamics. The bottom up models employed instead, ignore the feedbacks from the interaction of the energy sector with the wider economy within which it operates.

The development of a modelling framework that encompasses the multi market equilibrium of top down models with an engineering consistent representation of power producing technologies constitutes a long-standing challenge in applied energy policy analysis since the hybrid CGE model of Alan Manne (1977)[27]. Many different approaches[28] have been employed to link bottom up and top down models and can be classified in two main categories:

(i) *Hard link approach*, that is, integrating both bottom-up and top-down features in a consistent modelling framework. Such an integrated framework is provided by the specification of market equilibrium models as mixed complementarity problems (see Cottle and Pang [1992], Rutherford [1995]).

(ii) *Soft-link or decomposition approach* where bottom-up and top-down models are run independently of each other (Böhringer & Rutherford (2008), Bergman [1990], Hudson and Jorgenson [1974]). In this case results from one model are fed into the other, and vice versa.

A characteristic example of the first category is in Böhringer (1998) where the electricity generating technologies are modelled as specific activities within a mathematical-programming representation of the electricity sector, which is embedded directly in a computable general equilibrium model. In particular his approach is based on the complementarity formulation of the general equilibrium problem while the representation of the electricity producing sectors is based on Koopmans (1951) activity analysis framework. The standard aggregate production functions (CES or CD) used in the model are replaced by a set of discrete Leontief technologies (fixed input/output vector).

Towards the same direction lies McFarland et al. (2004) [EPPA model], who suggest a more flexible format through a CES representation of energy technologies. Their approach consists of splitting the energy sector using engineering bottom up data and then calibrate the model's smooth production functions on these data. In particular in their approach the cost estimates on capital, labour, and fuel inputs are used directly as the CES share parameters. The nesting scheme of the production function allows for the appropriate input substitution while the control of technology penetration rate is based on an endogenous quasi fixed factor coefficient introduced at the top level of the CES production function. Each technology produces electricity through a CES aggregation of its primary and

[27] ETA – Macro model where the process analysis ETA sub model of the U.S. energy system was linked with a one sector macro-model of the U.S. economy in a non linear optimization framework

[28] Jochem 1999, Muller 2000, Kemfert [1]). Messner and Schratenholzer, Koopmans and Willem te Velde 2001, Arikan and Kumbaroglou 2001.

secondary inputs (low elasticities of substitution chosen at this nesting level), while total electricity production results from a CES aggregation of all power technologies represented in the model (high elasticities of substitution at this nesting level).

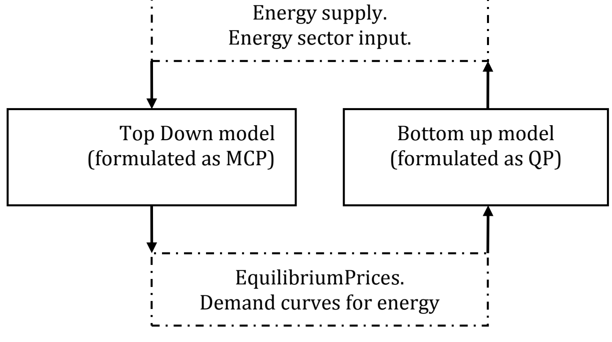
A disadvantage of this approach lies in its treatment of investment decisions. That is, investment is either allocated to electricity technologies exogenously or decided at the level of the aggregate electricity sector and then allocated to each technology using a logit function. This investment formulation although it allows for multiple technologies with different costs to coexist is not sufficient to represent the investment behaviour of the electricity sector (i.e. each sector should decide the level of investment as a function of its profit function and then this investment demand should be translated to demand for investment products produced by other sectors). In addition the non-smooth (kinked) representation of power supply results in sharp shifts in the technology mix of electricity production implying unrealistic swift switching between technologies.

The second category refers mainly to a decomposition method that links bottom up models with top down by combining different mathematical formats – mixed complementarity and mathematical programming. In Böhringer & Rutherford (2008) mixed complementarity methods (MCP) are used to solve the top-down economic equilibrium model and quadratic programming (QP) to solve the underlying bottom-up energy supply model. Then they reconcile equilibrium prices and quantities between both models through an iterative procedure (Figure 14) portray this iterative solution process).

Hybrid Bottom Up Top Down (BUTD**)** CGE models are still rare in the policy modelling literature due to difficulties arising from the integration of macroeconomic and engineering data in a consistent way. E3M-Lab has designed and incorporated into the GEM-E3 model a bottom up top down module. The motivation for this development was the need for a better representation of the electricity sector investment decision.

Toward this end electricity producing technologies were treated as separate production sectors while their investment decision is discrete. The advantage of this approach is that it is fully consistent with the general equilibrium framework while it leads to a full identification of the technologies. The rest of this section provides details on the exact formulation of the newly incorporated electricity producing sectors and on the reconciliation of engineering and input output economic data.

Figure 14: Iterative decomposition algorithm suggested by Böhringer & Rutherford (2008).



The bottom-up representation of the electricity sector extends the work performed within the DYN-GEM[29] project. The development of the database on generation costs, technology market shares and share of transmission and distribution cost to total cost of electricity production has been based on the TECHPOL database, the ENERDATA database and the PRIMES model database[30]. The technologies incorporated in the GEM-E3 model are presented in Table 11.

Table 11: Electricity producing technologies represented in GEM-E3 model

No	Description	No	Description
1	Coal fired	6	Hydro electric
2	Gas fired	7	Wind
3	Oil fired	8	CSP and Photovoltaics
4	Nuclear	9	Coal CCS
5	Biomass	10	Gas CCS

Electricity producing technologies are characterised by different cost structures and conversion efficiencies. The projections about capital, labour and fuel costs are substantially important since they influence the degree of use of each technology in power generation.

Generation costs are conceived in three categories: i) investment costs, ii) operating and maintenance costs and iii) fuel costs. Unit cost data and projections to the future for the first two categories were extracted from the TECHPOL and PRIMES database. The fuel costs depend on other variables of the GEM-E3. The data for each technology as introduced in the model are presented in Table 12.

The shares of each technology in power generation in the base year are introduced from energy balance statistics. Some of the potential technologies that may develop in the future are not used in the base year. Since the production function for power generation is calibrated to the base year, it is necessary to introduce artificially small shares even for the non-existing technologies in order to allow for the possibility of their penetration in the future under market conditions.

[29] The Dynamics of Innovation and Investment and its Impact on Policy Design in Energy and Environment for a Sustainable Growth in Europe, DYN-GEM-E3.

30 The Primes model database is not available to the public

Table 12: Electricity production cost shares

	Coal fired	Oil fired	Gas fired	Nuclear	Biomass	Hydro	Wind	PV
Agriculture					25.0			
Coal	24.3							
Oil		70.6						
Gas			73.2					
Chemicals				6.7				
Other Equipment Goods	5.0	0.5	0.5	0.5	1.5	1.0	9.8	0.8
Construction	3.0	2.0	4.7	1.0	1.5	3.0	5.8	6.7
Capital	56.6	22.3	19.3	87.6	67.4	80.3	80.0	83.2
Labour	11.1	4.7	2.2	4.2	4.6	15.7	4.4	9.2
Total	100	100	100	100	100	100	100	100

Source: Calculations based on TECHPOL and PRIMES databases.

Reconciliation of Input-output and Bottom-up Electricity Data

The Input-Output tables represent the electricity sector as an aggregate of two activities, the power generation and the transmission and distribution of electricity. This detail is not sufficient for the development of the bottom up model, so it has been necessary to split the Input-Output column and row in different activities, some corresponding to power generation by technology and the rest corresponding to transmission and distribution of electricity. The split was performed by combining data from energy balances and company-related economic data on generation and transmission and distribution activities by country. The aggregate data were based on Eurostat, IEA and USA DOE statistics[31].

In order to disaggregate the power sector appropriate mapping has been specified between the entries of the Input-Output table and the engineering information retrieved from the technical databases. For this purpose data on capital cost, fixed operating and maintenance cost, fuel cost and other variable operating and maintenance costs, related to the energy producing technologies to be incorporated in the model following cost elements have been extracted from the engineering database.

The unit costs have been associated with the corresponding cost elements of the Input-Output statistics, according to the following principles: i) annualised capital costs correspond broadly to operating surpluses, ii) fuel costs correspond to the fuel input, iii) fixed operating and maintenance cost correspond to non-energy inputs (materials), iv) variable operating and maintenance costs are associated with wages and salaries paid to employees in power generation.

Since the entire GEM-E3 model is calibrated on the social accounting matrices the macroeconomic data have been kept constant and the market and cost shares of the technologies have been appropriately adjusted. The purpose of the calibration has been to depart as little as possible from the flows suggested by the engineering information while

[31] For example, the disaggregation shows that the generation cost accounts for over half of total cost and in most E.U. countries they account for over 60% while transmission costs range between 5% and 10%

respecting exactly the totals appearing in the original input output table. For this purpose a cross entropy method has been applied[32].

The formulation for the power technologies, used in the GEM-E3 model, (as presented in the previous section) allows for no substitution between different power technologies and is expressed in a Leontief form with constant shares of the power mix.

[32] This calibration technique cannot be applied uniformly since each country has specificities that must be respected. For example there are cases where the input output data do not register a flow from agriculture to electricity (biomass fuel), or the engineering data suggest such capital allocations that lead to unrealistic investment to capital ratios by technology. Adjustments of data were made in order to cope with these difficulties.

Annex IX Energy efficiency

Introduction and implementation

This section shows how the firms and the households perform energy efficiency improving investments. In the previous versions of the model energy efficiency improvements were realized either exogenously without cost or endogenously through factor substitution. In the new setup the option to impose energy efficiency standards has been added. Towards this end energy efficiency cost curves have been added. The energy efficiency cost curves have been calibrated to estimates extracted from EMF(25) and from the relevant literature (i.e. Jakob, 2006).

In the efficiency module developed for GEM-E3 model households and firms invest to improve efficiency of energy use which means that the economy substitute materials (equipment, insulation, etc.) and services (e.g. provided by technicians for installation) for energy. The economic agents that undertake energy saving investments are the 14[33] representative firms (as depicted below) and one representative household in each region.

Table 13: GEM-E3 activities that undertake energy efficiency measures

No.	Economic Activity	No.	Economic Activity
1	Agriculture	9	Other Equipment Goods
2	Ferrous metals	10	Consumer Goods Industries
3	Non-ferrous metals	11	Construction
4	Chemical Products	12	Transport (Air)
5	Paper Products	13	Transport (Land)
6	Non metallic minerals	14	Transport (Water)
7	Electric Goods	15	Market Services
8	Transport equipment	16	Non Market Services

The amount of investment on energy saving technology is exogenous. It is assumed that the investment expenditure produce results one period after it takes place and continuously for a period of at least 20 years. The purpose of the investment concerns only the reduction of the unit consumption of energy in the sector or energy use of households, in which the investment takes place. That is, in the new setup agents use part of their income to acquire goods and services that are used to improve their energy efficiency. These goods and services accumulate to an energy saving capital stock that provides permanent energy efficiency improvements (with a declining/depreciation rate). The investment of a firm in energy saving equipment/capital increases energy efficiency and reduces its energy bill but it does not increase its productive capacity (i.e. it does not add to the capital stock of the firm). Energy efficiency improvement translates to additional demand for goods and services such as equipment goods, electrical goods, construction, market services (in fixed proportions). Similarly for households the expenditures on goods and services to improve

[33] Energy and electricity sectors are excluded.

their energy efficiency do not increase directly their utility, only indirectly through the energy bill reductions. Hence there are no direct effects on productive capacities or the consumption of other commodities. Of course indirect effects do exist and are quantified through the model. Finally it should be noted that the energy efficiency improvements are modelled so as to exhibit decreasing marginal returns (saturation effect).

To enforce the energy saving scheme to be implemented by firms and households the following methodology was adopted: The government raises an energy tax (proportional to the energy consumption of each economic agent). It imposes that rate of taxation to all consumers (firms and households) of energy, which is exactly necessary for collecting revenues equal to the amount of the energy saving expenditure. These revenues, given by equation [185], are then used by the government to finance the energy saving expenditures, ensuring public budget neutrality. Essentially the Government is used in the model to reallocate firms and households funds from their “optimum” placement in the reference case to the particular energy saving expenditures.

The revenues from the energy tax are:

$$tx\_effix_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot \sum_{pret} IOV_{pret,pr,er,t} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pr,er,t} \qquad [185]$$

The introduction of energy efficiency cost curves into the GEM-E3 model involves three tasks:

i) specification of the energy efficiency cost curve,
ii) calibration of the curve,
iii) Implementation within the current GEM-E3 model setup.

Specification of the energy efficiency cost curve

The main features of the energy efficiency cost curve are:

i) It is upper bounded (i.e. maximum energy efficiency improvement should be ~ 50%)
ii) The first available options for energy efficiency improvements are low cost
iii) Saturation effect (decreasing marginal returns)

The functional form that represents best the features of the energy efficiency cost curve is the logit function. The specific energy efficiency cost curve incorporated in the GEM-E3 model is given by [186] for firms and by [187] for households.

$$EFFI\_F_{br,er,t} = upper\_effi\_f_{br,er} \cdot \left(1 - e^{-\left[\frac{EFFI\_STOCK_{br,er,t}}{\sum_{pret} IOV_{pret,br,er,t,} + effi\_f\_x0_{br,er}}\right] \cdot speed\_effi\_f_{br,er}}\right) - effi\_f\_y0_{br,er} \qquad [186]$$

where:

$EFFI\_F_{br,er,t}$: the energy efficiency improvement rate (variable),

$upper\_effi\_f_{br,er,t}$: the upper bound of efficiency improvement level (calibrated parameter on extraneous data),

$EFFI\_STOCK_{BR,ER,T}$: the stock of the energy efficiency level (variable),

$speed\_effi\_f_{br,er}$: speed going to the inflexion point (calibrated parameter),

$effi\_f\_y0_{br,er}$: firms base year energy efficiency level (calibrated parameter),

$effi\_f\_x0_{br,er}$: firms base year energy efficiency level (calibrated parameter),

$IOV_{pret,br,er,t}$: the intermediate demand for energy products (variable)*:*

$$EFFI\_H_{er,t} = upper\_effi\_h_{er} \cdot \left(1 - e^{-\left[\frac{EFFI\_STOCK\_H_{er,t}}{\sum_{dg}\sum_{lnd} LLNDC_{lnd,dg,er,t} + effi\_h\_x0_{er}}\right] \cdot speed\_effi\_h_{er}}\right) - effi\_h\_y0_{er} \quad [187]$$

where:

$EFFI\_H_{er,t}$: the households energy efficiency improvement rate (variable),

$upper\_effi\_h_{er,t}$: the upper bound of efficiency improvement level (calibrated parameter on extraneous data),

$EFFI\_STOCK\_H_{er,t}$: the stock of the energy efficiency level (variable),

$speed\_effi\_h_{er}$: the speed going to the inflexion point (calibrated parameter),

$effi\_h\_y0_{er}$: the parameter related to households base year energy efficiency level (calibrated parameter),

$LLNDC_{lnd,pr,er,t}$: the consumption of linked non-durable goods.

The base year energy efficiency level parameters are computed from[188] and [189]

$$effi\_f\_y0 = u^f - by\_eel \quad [188]$$

where

by_eel : is the base year energy efficiency level (this is derived from extraneous data sources, the values used in the current version of the model are presented in Annex V of this report).

u^f : the upper level of energy efficiency parameter

$$effi\_f\_x0 = \frac{-\log\left(1 - \frac{by\_eel}{u^f}\right)}{s} \quad [189]$$

Data on energy efficiency

Energy efficiency cost estimates from Jacobi (2006) and EMF(25) have been used in order to calibrate the GEM-E3 energy efficiency cost curve[34]. The energy efficiency cost curves derived from the EMF(25) include both the commercial and the residential sector. The report provides market shares and efficiency levels for a number of electrical goods (Table 14) but not for buildings.

Table 14: Main categories of electrical goods covered by EMF (25)

No.	Commercial	No.	Residential
1	Refrigerated beverage vending machines	1	Refrigerators
2	Refrigeration equipment	2	Waterheaters
3	Unitary ac	3	Furnances and boilers
4	Terminal air conditioners and heat pumps	4	Cooking products
5	Clothes washers		
6	Waterheater & boiler		
7	Distribution transformers		
8	Small electric motors		

The available information from EMF(25) allows to compute the % price increase for each % saved in Kwh .

Table 15: Energy efficiency cost curve (electrical goods)

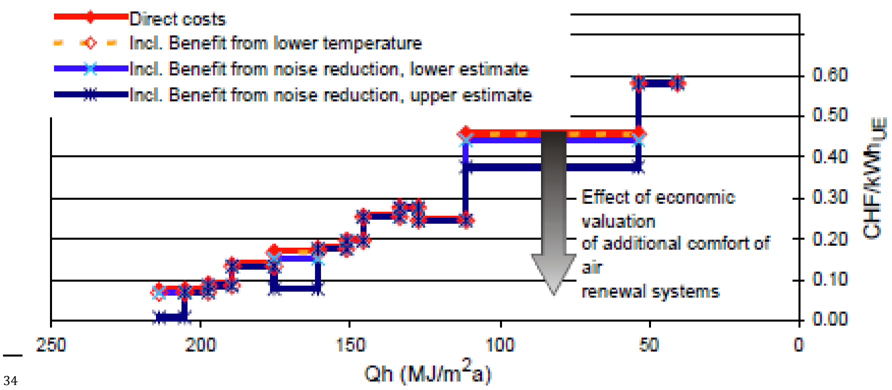
% Energy saved in Kwh	Increase in price
5%	0,57%
10%	1,60%
18%	5,01%
23%	8,24%

Source: Computations from EMF (25).

Data on energy efficiency measures for buildings (incl. estimates for co-benefits) have been extracted from Jakob (2006).

Figure 15: Marginal cost curve case study with oil heating

Source: Jakob (2006).



[34] … nmercial and residential sector) and Switzerland (Jakob(2006) buildings).

Energy efficiency in GEM-E3

The first step to incorporate the energy efficiency cost curve in the GEM-E3 setup is to introduce an additional factor namely the stock of energy saving technology. The stock of energy saving capital $EFFI\_STOCK_{br,er,t}$, $EFFI\_STOCK\_H_{br,er,t}$ is created by the accumulation of the goods on energy savings ([193],[[194]for firms and households respectively). The expenditure of firms and households on energy efficiency is given from equations [190], [191]. This expenditure depends on the energy tax imposed by the government, $tx\_effix_{pr,er,t}$ and $tx\_effi\_h_{pr,er,t}$ for firms and households respectively.

$$EFFI\_FLOW_{pr,er,t} \cdot PINV2_{pr,er,t} = tx\_effix_{pr,er,t} \cdot \sum_{pret} IOV_{pret,pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \quad [190]$$

$$\sum_{pr}\left(PINVP_{pr,er,t} \cdot EFFI\_FLOW\_H_{pr,er,t} + nrgeffi\_bcap\_h_{pr,t}\right) = \sum_{pret}\left(tx\_effi\_h_{pret,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot HCV_{pret,er,t}\right) \quad [191]$$

where:

$PINV2_{PR,ER,T}$: Price of investment goods used in energy efficiency,

$nrgeffi\_bcap\_h_{pr,br,t}$: the fixed factor coefficient of materials and services required to build the energy saving equipment (for the households)

Expenditure of firms on energy efficiency technologies is transformed into demand for goods of specific sectors according to the following equation:

$$IMAT\_FLOW_{pr,br,er,t} = EFFI\_FLOW_{br,er,t} \cdot nrgeffi\_bcap_{pr,br,t} \quad [192]$$

where:

$nrgeffi\_bcap_{pr,br,t}$: the fixed factor coefficient of materials and services required to build the energy saving equipment (in the model the coefficient is identical by industry)

Equations [193], [[194] provide the motion equation of the energy saving capital stock (for firms and households respectively):

$$EFFI\_STOCK_{br,er,t} = \left(1 - dloss_{br,er,t}\right)^{PERIOD} \cdot EFFI\_STOCK_{br,er,t-1} + \left(\frac{1-\left(1-dloss_{br,er,t}\right)^{PERIOD}}{dloss_{br,er,t}}\right) \cdot EFFI\_FLOW_{br,er,t} \quad [193]$$

$$EFFI\_STOCK\_H_{er,t} = \left(1 - dloss_{er,t}\right)^{PERIOD} \cdot EFFI\_STOCK\_H_{er,t-1} + \left(\frac{1-\left(1-dloss_{br,er,t}\right)^{PERIOD}}{dloss_{br,er,t}}\right) \cdot EFFI\_FLOW\_H_{er,t} \quad [194]$$

where:

$EFFI\_STOCK_{br,er,t}$, $EFFI\_STOCK\_H_{br,er,t}$: the stock of energy saving technology,

$dloss_{br,er,t}$: the decay parameter for the energy efficiency improvements,

PERIOD: the time between two GEM-E3 runs (usually five).

Then energy productivity (*tge*) is formulated as a positive function of the stock of energy saving technology.

$$tge = tge + effi \qquad [195]$$

It is assumed that there is a time lag between the expenditure and the realisation of the efficiency gains. Currently this is modelled as a one period lag. The expenditure on energy efficiency, either from Households or Firms is translated to demand for certain goods and services in fixed factor proportions (the exact shares for each category are presented in Table 16.

Table 16: Sector contribution to energy efficiency investment

	Expenditure, in percent of total
Electric Goods	20%
Construction	70%
Market Services	10%

Accordingly the price of the investment good is given below.

$$PINV2_{br,er,t} = \sum_{pr} \left(PINVP_{pr,er,t} \cdot nrgeffi\_bcap_{pr,br,t} \right) \qquad [196]$$

The energy efficiency investment is financed through a tax neutral instrument. That is a tax on energy consumption is imposed in households and firms. Then government uses these revenues so as to perform the energy efficiency investment and provide households and firms with the respective energy efficiency improvement. Hence the household price and the firms' user cost of energy become:

$$PHC_{pr,er,t} = \left(1 + txit_{pr,er,t}\right) \cdot \left(1 + txvat_{pr,er,t}\right) \cdot PY_{pr,er,t} + tx\_effi\_h_{pr,er,t} \cdot \frac{PCI_{er,t}}{PCIBASE_{er,t}} \qquad [197]$$

$$PEUPR_{pr,br,er,t} = \begin{cases} \left[\begin{array}{c} PIO_{pr,er,t} + TXENGR_{pr,sr,er,t} \cdot \dfrac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot AER_{pr,br,er,t} \\ + \sum_{poem} \left[\begin{array}{c} TXENV_{poabe,br,er,t} \cdot bec_{poabe,br,er,t} \\ \cdot aer_{pr,br,er,t} \end{array} \right] \\ + tx\_effix_{br,er,t} \cdot \dfrac{PCI_{er,t}}{PCIBASE_{er,t}} \end{array} \right] \\ for\ PR = PRE \\ \left[\begin{array}{c} PIO_{pr,er,t} + TXENGR_{pr,sr,er,t} \cdot \dfrac{PCI_{er,t}}{PCIBASE_{er,t}} \cdot AER_{pr,br,er,t} \\ + tx\_effix_{br,er,t} \cdot \dfrac{PCI_{er,t}}{PCIBASE_{er,t}} \end{array} \right] \\ for\ PR = PRENEM \end{cases} \qquad [198]$$

where:

$TXENGR_{pr,sr,er,t}$: the energy tax per sector,

$TXENV_{poem,br,er,t}$: the environmental tax,

$bec_{poabe,br,er,t}$: the emission coefficient per monetary unit.

$aer_{pr,br,er,t}$: the share of energy consumption with emission.

Investment now becomes:

$$INV_{se,er,t} = \begin{cases} \sum_{pr}\left(tcinv_{se,pr,er,t} \cdot PINV_{pr,er,t} \cdot INVV_{pr,er,t}\right) & for\ se = 1, \\ \sum_{pr}\left(tcinv_{se,pr,er,t} \cdot PINV_{pr,er,t} \cdot INVV_{pr,er,t}\right) & for\ se = 2, \\ \sum_{pr}\left(tcinv_{se,pr,er,t} \cdot \left(PINV_{pr,er,t} \cdot INVV_{pr,er,t}\right)\right) + \sum_{pr} PINV2_{pr,er,t} \cdot EFFI\_FLOW_{pr,er,t} & \\ \quad + \sum_{pr} PINVP_{pr,er,t} \cdot EFFI\_FLOW\_H_{pr,er,t} \cdot nrgeffi\_bcap\_h_{pr,t} & for\ se = 3, \\ \sum_{pr}\left(tcinv_{se,pr,er,t} \cdot PINV_{pr,er,t} \cdot INVV_{pr,er,t}\right) & for\ se = 4, \end{cases} \quad [199]$$

where:

$tcinv_{pr,er,t}$: the share of each institutional sector in total investment

Annex X Indicators of Energy Security

The attempt to develop energy security indicators is inevitably affected from the ubiquity of complexity in energy production, supply and demand as well as from the complex inter-relationships between different types of energy. The energy security indicators discussed in the literature to date range from simple to more complex ones. Simple indicators focus on quantity or are indicators that can be expressed in physical or monetary terms. Complex or aggregate indicators take into consideration several dimensions of energy security (context, diversity, availability etc). The following subsections briefly review several indicators suggested in the literature to date.

Selected energy security indicators

The energy security price index (ESPI)

Developed by Lefevre (2010), the ESPI index is designed to take into account factors such as diversification of fuel mix in energy supply, political stability and market concentration to measure energy security. Each exporting country is assigned a risk factor which consists of the country's political risk rating (r_c, ranging from 1 to 3) and its market power in the global fuel market (ω_{cf}). The global risk factor ($ESMC_{pol\text{-}f}$) to be used as a weight in the calculation of the ESPI index is simply the sum of the individual risk factors:

$$ESPI = \sum_f \left(\frac{E_f}{TPES}\right) ESMC_{pol-f}$$

and

$$ESMC_{pol-f} = \sum_c (r_c \omega_{cf}^2)$$

where $\frac{E_f}{TPES}$ is the share of fuel *f* in total primary energy supply in the examined country,

r_C is the political risk rating of export country *c* ranging from 1 (low risk) to 3 (high risk),

ω_{cf} denotes the share of export country *c*'s net export potential in global export potential of fuel *f* (in percentage points).

Rating of risk scales up Herfindahl's concentration index and $ESMC_{pol\text{-}f}$ is high when few high-risk exporters dominate the world market.

Frondel and Schmidt index

The index proposed by Frondel and Schmidt (2008) quantifies the degree of a country's reliance on fossil fuel imports. The indicator is a weighted average of fuel-specific risks, with the weights being the share of imports by country of origin in the total energy supply of the specific fuel.

Denoting the probability of supply disruptions in export country *j* by r_j , the authors suggest the following quadratic form of measurement of a nation's supply risk related to fuel:

$$risk_i := x_i^T R x_i = x_{id}^2 r_d + \sum_{j=1}^{J} x_{ij}^2 r_j$$

where,

$x_{id} + x_{i1} + \ldots + x_{ij} = 1, \qquad i = 1, \ldots, I$

is the share of imports by country of origin in total supply of fuel *i* (*d* stands for domestic contribution)

The risk factor assigned to a country's own contribution to domestic supply can be assumed to equal zero: $r_d = 0$.

The estimation of the probability of an unexpected interruption in individual export countries is based on OECD classifications. The risk factor calculation allows taking into consideration the possibility of cartels (like OPEC) in the energy market and hence the correlation of supply disruptions among the cartel member countries. The index can be generalized so as to measure the country's security over all kinds of energy sources. In this case the indicator is able to account for correlations between supplies of different fuels.

The composite energy security index (CESI)

CESI is a combination of the energy security price index (ESPI), net energy imports to total energy consumption and energy intensity indices. The index takes into consideration the net import dependency on specific fuels, the significance of energy in the economy and the price risks associated with specific fuels. The index is specified as follows:

$ESI = \sum_f (\frac{M_f - X_f}{GDP}) ESMC_{pol-f}$ for all f where $M_f > X_f$

where M_f imports and X_f exports. This indicator represents an aggregation of the fuel-specific energy security market concentrations (including political risks) weighted by the shares of (positive) net imports of the respective fuels in GDP (Böhringer and Keller, 2011).

Jansen's index

Jansen et al (2004) elaborate the Shannon index of diversity into four indicators of long-term energy supply security.

The indices are designed so as to take into account different aspects of energy security, stepwise, starting from a simple indicator (I_1) of energy security which depends on fuel diversification in primary energy supply. The last index (I_4) measures energy security as a function of import dependence and diversification, political stability and depletion of reserves.

The indicators take values higher than one, suggesting high diversity in fuel supply variety and balance. The first energy security indicator is given by:

$$I_1 = -\sum_i (c_i^1 p_i lnp_i)$$

where I_1 is energy supply security indicator, p_i *is* the share of primary energy source *i* in total primary energy supply, *i = 1…M* is primary energy source index and c_i^1 is the correction factor to p_i for indicator I_1The rest of the indicators are variants of this first one, by elaboration on the correction factor.

The second indicator I_2 results from an adjustment of the basic indicator so as to account for net import dependency. The indicator is calculated as follows:

$$I_2 = -\sum_i (c_i^2 p_i lnp_i)$$

subject to:

$$c_i^2 = 1 - m_i(1 - \frac{S_i^m}{S_i^{m,max}})$$

where c_i^2 is the correction factor to p_i for indicator I_2, m_i is the share of net import in primary energy supply of fuel *i*, S_i^m is the Shannon index given by:

$$S_i^m = -\sum_j (m_{ij} lnm_{ij})$$

where m_{ij} is the share of imports from region *j* in total imports of fuel *i*. $S_i^{m,max}$ denotes the maximum value of Shannon index.

The third indicator accounts for the level of long-term political stability in regions of origin. In this step, the authors suggest that the UNDP Human Development Indicators should be employed. The third indicator is formulated as follows:

$$I_3 = -\sum_i c_i^3 \, p_i \ln p_i$$

where:

$$c_i^3 = 1 - m_i\left(1 - \frac{S_i^{m*}}{S_i^{m*,max}}\right)$$
$$S_i^{m*} = -\sum_j h_j \, m_{ij} \ln m_{ij}$$

h_j is the parameter measuring political stability in region *j*, ranging from 0 (extremely unstable) to 1 (stable), S_i^{m*} is the Shannon index of import flows of resource *i*, adjusted for political stability in the regions of origin, $S_i^{m*,max}$ is the maximum value of the Shannon index.

The fourth indicator allows for the level of resource depletion on an additional basis.

$$I_4 = -\sum_i c_i^4 * p_i * \ln p_i$$

where:

$$c_i^4 = \{1 - (1 - r_{ik}) * (1 - m_i)\} * \{1 - m_i * \left(1 - \frac{S_i^{**}}{S_i^{**,max}}\right)\}$$

$$r_{ij} = Min\left\{\left[\frac{\left(\frac{R}{P}\right)_{ij}}{50}\right]^a ; 1\right\}, for\ a \geq 1$$

$$S_i^{m**} = -\sum_j (r_{ij} * h_j * m_{ij} * \ln m_{ij})$$

r_{ij} is a depletion index for resource *i* in import region *j*, r_{ik} is the depletion index for resource *i* in home region *k*, for which the indicators are determined, $\left(\frac{R}{P}\right)_{ij}$ is the proven reserve-production ratio for resource *i* in region of origin *j.*

While the fourth index developed by Jansen et al (2004) captures several parts of the energy security concept, it has been criticized due to the lack of robust ground on the balance between different elements (fuel diversity, import dependence/diversity, political stability and depletion) and on the arbitrary results that may result from the latter (IEA, 2007a).

Supply/Demand side indicators

Supply/Demand index (S/D)

Scheepers et al (2007) propose a S/D index which extends beyond security of supply and considers the full spectrum of the energy system: supply, final demand, energy conversion and transport.

The simplest arithmetic form of this index is:

$$\frac{S}{D} = weight\ of\ demand \cdot demand\ value\ +\ weight\ of\ supply \cdot\ supply\ value$$

where demand and supply values are sub-indexes of demand and supply values resulting from simple functions of factors like shares of, among others, supply origins, efficiencies, reserve factors, network capacity, refinery and storage capacity. Functions are kept simple in favour of transparency. Demand, supply and factors are weighted on the basis of expert judgments.

Oil vulnerability index

Studies on energy security have paid special attention to oil vulnerability i.e. the exposure of oil consuming countries to volatile oil prices and to abrupt disruptions of oil supply (CIEP, 2004; INDES, 2004; IAEA, 2005; APERC, 2003). Three major risks that contribute to the overall oil vulnerability of an economy have been suggested: market (or economic) risk, supply risk and environmental risk.

Market risk of an economy refers to the risks of macroeconomic effects due to price fluctuations in oil markets. Supply risk of an economy refers to the risks of physical disruptions in oil supplies. The environmental risk relates to climate change, global warming, accidents and polluting emissions due to increased oil usage.

Gupta (2008) combines several indicators (like the ratio of value of oil imports to GDP, oil consumption per unit of GDP, GDP per capita and oil share in total energy supply, ratio of domestic reserves to oil consumption, exposure to geopolitical oil market concentration risks as measured by net oil import dependence, diversification of supply sources, political risk in oil-supplying countries and market liquidity) using the principal component technique in order to derive a composite index which captures the relative sensitivity of various economies towards developments of the international oil market. In the constructed index a higher value indicates higher vulnerability.

The index is formulated as follows:

$$OVI_k = b_1X_{1k}+b_2X_{2k} +...+b_nX_{nk} + e$$

Where $\mathrm{OVI_k}$ is the oil vulnerability index of country *k,* X_{nk} is the set of indicators used so as derive the composite index and *e* the error term.

Resource estimates

Resource estimates quantify the existence of energy resources and their future availability. Several authors have suggested using resource estimates as indicators of energy supply (see for instance Kruyt et al, 2009). In this case the remaining reserves of energy sources can be used as a direct energy security indicator.

Reserves to production ratios

The reserves to production ratios $(\frac{R}{P})$ indicate the years of production left at current production levels (Feygin and Satkin, 2004). Neither reserves nor production rates are fixed, thus their combination is also a dynamic quantity. In practice, constant factors are usually used for both. Projected production levels can be used instead of current ones (dynamic reserve to production ratio), but in this case problems arise with the transparency of the obtained indicator.

Diversity indices

Diversity indices quantify diversity in energy (fuel) type, geographical source and energy supplier. Stirling (1999) suggests that diversity indexes should consider:

- Variety
- Balance and

- Disparity

Given the difficulty to define disparity it has been difficult to practically define such indices (Kruyt et al, 2009). For diversity of order one a formulation of this index would be:

$$D^1 = \exp(\, S_i^m)$$

where S_i^m is the Shannon index. In the absence of an appropriate measure of disparity, the indices that measure only two of the three key elements of diversity are formally called "dual concept" indices. Concerns on these indicators are raised from the fact that the categorization of options influences the results of these indices, and the lack of an objective measure of disparity may render them subjective or arbitrary.

Import dependence indicators

Import dependence indicators quantify the degree of dependence on energy imports and can be expressed either in physical or in monetary terms (Alhajji and Williams, 2003). A formulation of this index is:

$$Import\ dependence = \frac{Energy\ imports}{Total\ energy\ consumtion}$$

For countries/regions being transport hubs a more realistic version of these indicators allows for subtracting the exported energy. In this case the indicator would be formulated as:

$$Import\ dependence = \frac{(Energy\ imports - Energy\ exports)}{Total\ energy\ consumtion}$$

Refined versions of import dependence indicators have also been developed and employed (see for instance APERC, 2007) for the combined measure of import dependence and diversification. At world level, international trade in energy carriers, energy trade and share of global demand that is traded internationally can also be employed as indicators of dependence (Kruyt et al, 2009).

Political stability indicators

These indicators quantify the political risk associated with countries suppliers of energy. These indicators make use of works that quantify governance and political stability like the International Country Risk Guide (see IEA, 2004), the World Bank Governance Indicators (IEA, 2007a) and the UNDP Human Development Indicators (Jansen et al, 2004). Governance and political stability indicators have been suggested to be employed in order to quantify the risk of energy security disruption due to political developments.

Energy prices

Oil prices can play a major role as an indicator of energy security with oil being the primary energy source in most countries. Prices can be directly employed as indexes of availability and affordability of energy (see Kruyt et al, 2009). A shortcoming of this approach is related to the fact that energy prices can also be affected by non-market factors (like speculation and short-term availability of resources).

Share of zero-carbon fuels

This indicator quantifies the share of renewables and nuclear in total primary energy supply (APERC, 2007) and is formulated as follows:

$$Share\ of\ zero\ carbon\ fuels = \frac{E_f}{TPES}$$

where E_f energy supply from renewables and nuclear and *TPES* total primary energy supply. Concerns on this indicator regard the appropriate consideration of carbon content and the acceptability of the indicator regarding other energy options like nuclear energy.

Market liquidity

Market liquidity indicator quantifies the market ability to cope with demand and supply fluctuations. IEA (2004) defines market liquidity indicator as the exponential function of the ratio of a country's consumption over the total of that fuel available on the market ($e^{\frac{1}{p_f}}$, where p_f is the total supply availability in the accessible market of fuel type *f)*. The concept of market liquidity is also linked to price elasticity. Datar (2000) suggests for stock markets, using a coefficient of elasticity of trading (CET) as an indicator of market liquidity defined as the relative change in trading volume over the relative change in price:

$$CET = \frac{\%\ change\ in\ trading\ volume}{\%\ change\ in\ price}$$

In this case values below unity would indicate an inelastic market, while values above unity would indicate elastic markets.

Demand-side indicators

Demand side indicators aim at quantifying the impact of energy shortages (Kruyt et al, 2009). Among others they include indicators like the energy or fuel intensity of the economy or households. Energy intensity indicator can be formulated as follows:

$$EI = \frac{E}{GDP}$$

where *EI* is energy intensity, *E* is total energy consumption and GDP is Gross Domestic Product. In this category are also included indicators related to energy expenditures where high energy expenditures are regarded as indicative of affordability and of the ability of securing energy supplies (Kendell, 1998). Energy expenditures (*EE*) index can be formulated as follows:

$$EE = \frac{EX_f}{TEX}$$

where EX_f are expenditures for energy type f (for instance expenditures for oil) and *TEX* are total expenditures on imported goods and services.

Indicators used in the GEM-E3 model

Based on the literature to date, the indicators used in the GEM-E3 model are:

Import dependence indicators: Import dependence indicators are employed in order to get an estimate of trade dependencies (the indicators are appropriately adjusted for countries serving as transport hubs). Higher import dependence is associated with higher cost of disruption and lower energy security.

Diversity indices: Diversity indices accounting for both fuels and suppliers are employed with the aim to derive a more transparent view of fuel and supplier dependence of energy importing countries. In this case higher fuel and supplier diversification are associated with higher energy security.

Demand side indicators: Demand side indicators like household and/or economy energy intensity are employed so as to quantify demand side dimensions of energy security and possible reactions resulting from changes in supply of energy sources.

Share of zero-carbon fuels and emissions' indicators**:** Indicators of shares of renewables in total primary energy supply and estimates on emissions quantify aspects of environmental sustainability of energy security. In the case of emissions, total greenhouse gas emissions from energy production (and use) are employed in order to quantify climate change impact and/or annual emissions of various pollutants to measure the pollution levels.

Reserves to production ratio: This indicator quantifies aspects of energy security availability and affordability.

Annex XI Stochastic version of GEM-E3

Introduction

This section presents the methodological approach adopted in order to enable the performance of sensitivity analysis on the results of the GEM-E3 model. The new setup of the GEM-E3 model provides the option to make all its parameters stochastic according to user defined probability distribution. Monte Carlo methods are employed in order to generate large number of pseudo samples. Since the variables of the GEM-E3 model include the stochastic input parameters, they also present a stochastic behaviour following an empirical joint probability distribution. Statistical analysis of these samples enables probabilistic statements on any function involving GEM-E3 variables. Furthermore the module allows for modifications in the distribution of input parameters thus producing alternative sets of results in the form of conditional distributions that can be used to test the robustness of changes arising from input modifications (sensitivity analysis of impacts).

Modelling methodology

Overview

In the new stochastic version of GEM-E3 the input parameters are stochastic. The productivities, elasticities, scale, share, rates and any other parameter included in the GEM-E3 model (see Table 17 – Table 20, respectively) have a stochastic representation. The parameters can be grouped in the following categories according to three main properties: i) constraint that they should sum up to a specific value i.e. shares ii) sign constraint i.e. for some parameters it is illegal to take both positive and negative values and iii) dimensionality.

Table 17: Stochastic GEM-E3 parameters – Productivities

No.	Productivities	Description
1	TGK	Technical Progress on Capital
2	TGL	Technical Progress on Labour
3	TGE	Technical Progress on Energy
4	TGM	Technical Progress on Materials

Table 18: Stochastic GEM-E3 parameters – Elasticities

No.	Elasticities	Description
1	S1	Substitution Elasticity in PF (K and others)
2	S2	Substitution Elasticity in PF (EL and others)
3	S3	Substitution Elasticity in PF (L MA and fuels)
4	S4	Substitution Elasticity in PF (Materials)
5	S5	Substitution Elasticity in PF (Fuels)
6	SIGMAX	Substitution Elasticity Armington (Domestic - Imports)
7	SIGMAI	Substitution Elasticity Armington between Countries
8	A1INV	Elasticity Delay Parameter in Investment Function

Table 19: Stochastic GEM-E3 parameters - Shares

No.	Shares	Description
1	DKAV	Scale Parameter Capital (Upper Level)
2	DLEM	Scale Parameter LEM Aggregate (Upper Level)
3	DEL	Scale Parameter Electricity (LEM Level)
4	DLMO	Scale Parameter LMO Aggregate (LEM Level)
5	DL	Scale Parameter Labour (LMO Level)
6	DE	Scale Parameter Fuel Aggregate EN (LMO Level)
7	DM	Scale Parameter Material (LMO Level)
8	DMPR	Scale Parameter Products (IO Level)
9	DELTA	Scale Parameter Armington (IMP XXD)
10	BETA	Scale Parameter Armington for Substitution among imports
11	A0INV	Scale Parameter of Investment Function
12	AC	Scale Parameter of Armington
13	DEPR	Scale Parameter Fuel (3d Level)

Table 20: Stochastic GEM-E3 parameters – Other parameters

No.	Others	Description
1	BHCFV	LES (lower level) Consumption Category Share Parameter
2	CHCFV	LES (lower level) Obliged Consumption (in volume)
3	DECLH	Depreciation Rate (Household)
4	THCFV	Share of Branch in Delivery to Private Consumption
5	TGCV	Share of Branch in Delivery to Public Consumption
6	GCTV	Public Consumption
7	GINVVEXO	Endogenous Public Investment
8	POP	Active Population
9	STGR	Growth Expectation in investment function

Let Y_t be the value of the parameter Y at time t. According to GEM-E3 specification; this parameter could have arguments related to country, institutional sector, activity, pollutants, emissions, and purpose of consumption etc. Based on a hierarchical scheme (described

below) and by using random numbers which are generated from the normal distribution we define the value of the parameter at time *t+1*, Y_{t+1}.
More formally,

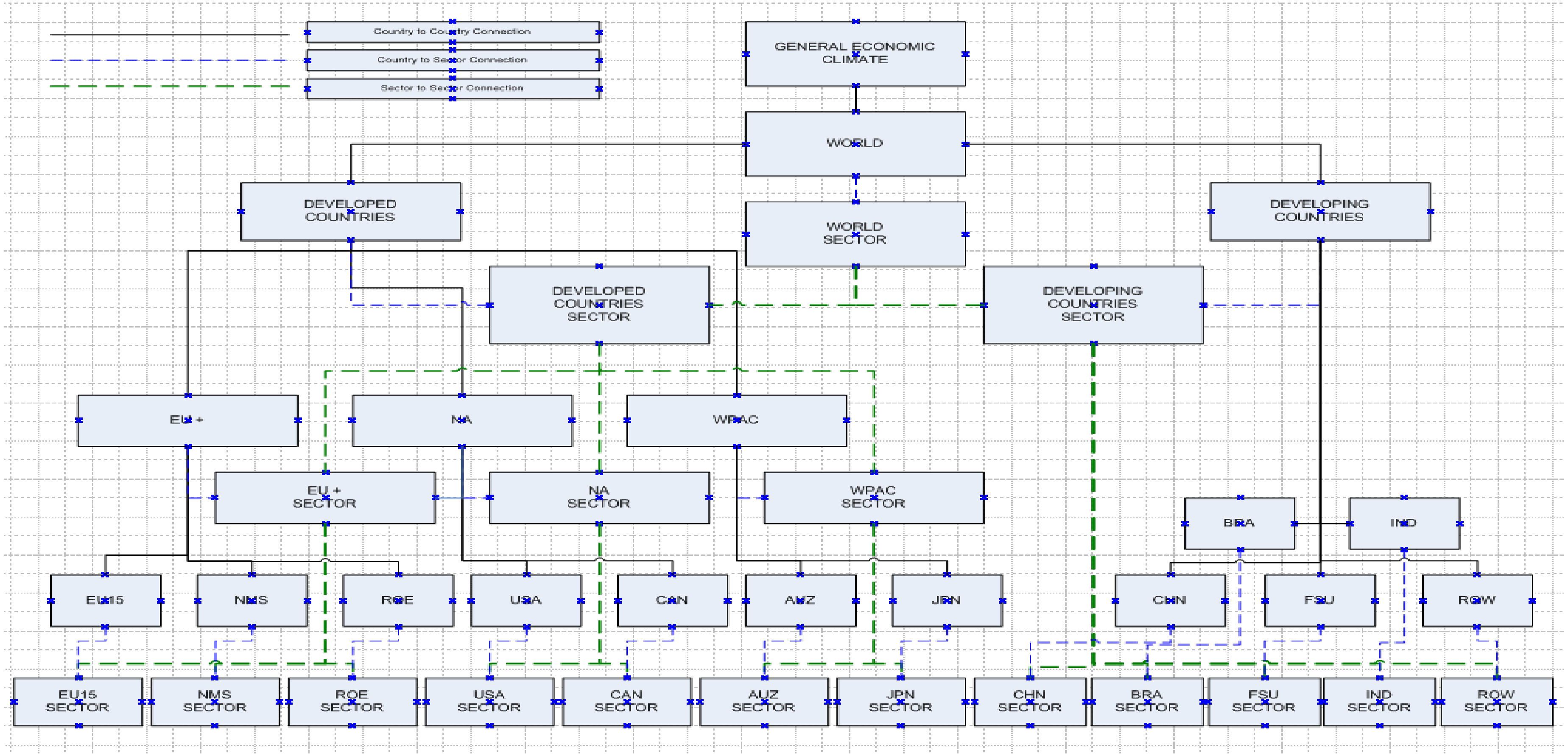
$$Y_{t+1} = Y_t \cdot e^{Z_{i+1}} \qquad [200]$$

Where *Z* is an independent variable with $Z_{t+1} \sim N(0, (\sum w_i \sigma_i)^2)$ and *i* defines the position (level) of each parameter on the hierarchical scheme. This hierarchical scheme provides the dependence/relation pattern of each parameter. For each level of the hierarchical scheme the respective weights are defined. Through this scheme the user can provide the correlation between the different parameters by selecting the appropriate weights.

Hierarchical Scheme

In this section the hierarchical scheme used to include a stochastic representation of the GEM-E3 is described. To follow the notation in GAMS code, a parameter which has arguments: sector (pr), region (cott) and time (stime) is described as *Y(pr,cott,stime)*. The hierarchical scheme employed is presented in Figure 16:

Figure 16: Stochastic model hierarchical scheme



In each year random numbers are generated from the normal distribution for all places in the hierarchical scheme, $\varepsilon_{i,t} \sim N(0, \sigma_i)$. Starting from the upper level (General Economic Climate, GEC), the lower levels with the upper levels are correlated as follows:

$$Z^{gec} = \varepsilon^{gec} \qquad [201]$$

$$Z_{cott}^{lower} = W_{cott}^{upper} Z_{cott}^{upper} + W_{cott}^{lower} \cdot \varepsilon_{cott}^{lower} \qquad [202]$$

where cott = { EU27 countries separately, USA, CAN, AUZ, JPN, BRA, IND, CHN, FSU, ROW,CRO, ANI} and

$$Z_{cott,pr}^{lower} = W_{cott,pr}^{upper} Z_{cott,pr}^{upper} + W_{cott}^{middle} \cdot Z_{cott}^{middle} + W_{cott,pr}^{lower} \cdot \varepsilon_{cott,pr}^{lower} \qquad [203]$$

where pr = {all sectors used in the model}.

Finally, by following the above process, we end up at the lowest level, with variates that are correlated, depending on the choice of the weights and the standard deviation of the εi,t.

The values of εi,t at the lowest level are normally distributed with zero mean and $\sum w_i \sigma_i$ standard deviation. The majority of parameters and variables of the GEM-E3 model are sign-restricted and hence the normal distribution that plays such a dominant role in statistical analysis cannot be indiscriminately applied. Therefore the Log Normal distribution is used as the default in the generation program. That is:

$$Y_{t+1} = Y_t \cdot e^{Z_{i+1}}$$

Based on the above hierarchical scheme similar schemes have been used in order to model parameters that are defined over different sets.

Alternative hypotheses for the hierarchical scheme could be done without changing the whole structure. For example, if we want a parameter to be independent across countries and sectors, we simply define the weights in equations [202] and [203] as zero except the lower weight $W_{cott,pr}^{lower}$. Similar if we want the parameter to have correlation only across country we define the upper weights in equation [203] as zero.

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Abstract
The computable general equilibrium model GEM-E3 has been used in a large set of climate policy applications supporting Commission policy proposals during the last decade, as well as in other environmental and economic policy areas. It can be considered a multi-purpose macroeconomic model, designed to estimate the effects of sector-specific policies on the economy as a whole.

The main purpose of this publication is to provide extensive documentation of the model's equations and its underlying databases, in order to offer to the broader audience an accurate description of the model characteristics.

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